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Know
dive

World models

Intensional representation

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Extensional and intensional representation

Observation 6.1 (World model, extensional representation) World models, as defined so far are **extensional representations** of the world, namely they are defined as sets of assertions a and facts f, plus an interpretation function IAwhich allows to define which assertions denote which facts in one or more reference models.

But what is a *fact*? How do we construct assertions about facts? The answer to this question requires defining an **intensional representation** of world models, namely the **representation mechanisms** which allow to construct assertions and facts starting from a finite set of primitive component elements.

Notation 6.1 (Extensional and intensional representation of a set) Let S be a set. Then by Se we mean the **extensional representation** of S, i.e., as a set of elements (e.g., facts, assertions, but not only); by Si we mean the **intensional representation** of S, where the elements of Se are defined intensionally, starting from a set of primitive components. The notation is dropped when no confusion arises.

Domain – Intensional representation

Intuition (Domain, intensional representation) The intensional representation of a domain is composed of three components, as follows

- *entities*, associated with those elements of the representation which can be isolated and distinguished from the rest;
- *classes* (sets) of entities, characterized by the fact they have some common characteristics which is not shared by the entities of the other sets;
- *relations* among entities, which collect multiple entities sharing a common property.

Does it fit your intuition? How you would describe the world?

Domains and facts, intensional representation

Definition 6.1 (Domain, intensional representation) The **intensional representation** Di **of a domain** D is defined as

$$Di =< E, \{C\}, \{R\} >$$

with

$$E = \{e\}, \qquad C \subseteq E, \qquad R \subseteq E \times \cdot \cdot \cdot \times E$$

where E = {e} is a set of **entities**, {C} is a set of **classes** of entities, {R} is a set of n-ary **relations** Rn, for some n. E is called the **universe** of Di or also the **universe of interpretation**.

Definition 6.2 (Fact, intensional representation) The **intensional representation** De **of a fact** f has one of the following four forms

$$e \in C, \qquad < e1, ..., en > \in R, \qquad C \subseteq E, \qquad Rn \subseteq C1 \times \cdot \cdot \cdot \times Cn$$

with e, ei ∈ E and C, Ci ⊆ E.

Data and Knowledge domains

Definition. (Domain, data, knowledge, mixed) A **data domain** contains only facts of the form

$$e \in C$$

$$< e1, \ldots, en > \in R.$$

A **knowledge domain** contains only facts of the form

$$C1 \subseteq C2,$$

$$Rn \subseteq C1 \times \cdot \cdot \cdot \times Cn.$$

A **mixed domain** contains all types of facts.

Data and Knowledge domains

Example 6.2 (Data domain)

sofia ∈ Person,
sofia ∈ Woman,
<paolo, rocky> ∈ HasDog
<sofia, paolo> ∈ Near
paolo ∈ Man <sofia

<rocky, paolo> ∈ DogOf
paolo ∈ Dog
rocky ∈ Dog
<rocky, sofia> ∈ DogOf
paolo> ∈ FriendOf

<paolo, sofia, stefania> ∈ Between

Data and Knowledge domains

Example 6.2 (Knowledge domain)

Person ⊆ Entity

Dog ⊆ Entity

Animal ⊆ Entity

Near ⊆ Entity × Entity

FatherOf ⊆ Person × Person

HasDog ⊆ Person × Dog

DogOf ⊆ Dog × Person

FriendOf2 ⊆ person × person × person

FriendOf1 ⊆ Person × Person

ChildOf ⊆ Person × Person

where Entity stands for E.

Assertional Language

Definition (Assertional language, intensional representation) The **intensional representation** LiA **of an assertional language** LA is defined as

$$\mathrm{L}iA = \langle E, \{C\}, \{P\} \rangle$$

where $E = \{e\}$ is a set of **(names of) entities**, *{C}* is a set of **concepts**, where a concept is a **name of a class**, *{P}* and a set of **properties**, where a property is a **name of a relation**.

Definition (Assertional language, extensional representation) The **extensional representation** LeA **of an assertional is** L$eA = \{a\}$ with a having one of the following four (five) forms

$C(e)$, $Pn(e1, \ldots, en)$, $C1 \leq C2$, $C1 \equiv C2$, $Pn(C1, \ldots, Cn)$

Interpretation function

Definition (Interpretation function, intensional interpretation) The **Intensional representation** IA **of an interpretation function** $IA : LA \rightarrow D$ of an assertional language is defined as

$$IA = < Ie, IC, IP >$$

with:

$$Ie : E \rightarrow E$$

$$IC : \{C\} \rightarrow \{E\}$$

$$IP : \{Pn\} \rightarrow \{E\} \times \cdot \cdot \cdot \times \{E\}$$

and such that:

$$IA(C(e)) = IC\ (C)\ (Ie\ (e)) = e \in C$$

$$IA(Pn\ (e1, . . . , en)) = IP\ (Pn)\ (Ie\ (e1), . . . , Ie\ (en)) = < e1, . . . , en > \in Rn$$

$$IA(C) = IC\ (C) = C \subseteq E$$

$$IA(Pn\ (C1, . . . , Cn)) = IP\ (Pn)\ (IC\ (C1), . . . , IC\ (Cn)) = Rn \subseteq C1 \times \cdot \cdot \cdot \times Cn$$

World model, intensional representation

Definition 6.10 (World Model, intensional representation) Given a **World Model**

$$W = \langle LA, D, IA \rangle,$$

its **intensional representation** Wi is defined as

$$Wi = \langle LiA, Di, IiA \rangle$$

World models, models and theories – The practice

1. Select the world model (crucial representation choice)

$$Wi =< L_{iA}, D_i, I_{iA}>$$

2. Agree on L_{iA}, I_{iA} (… and therefore D)

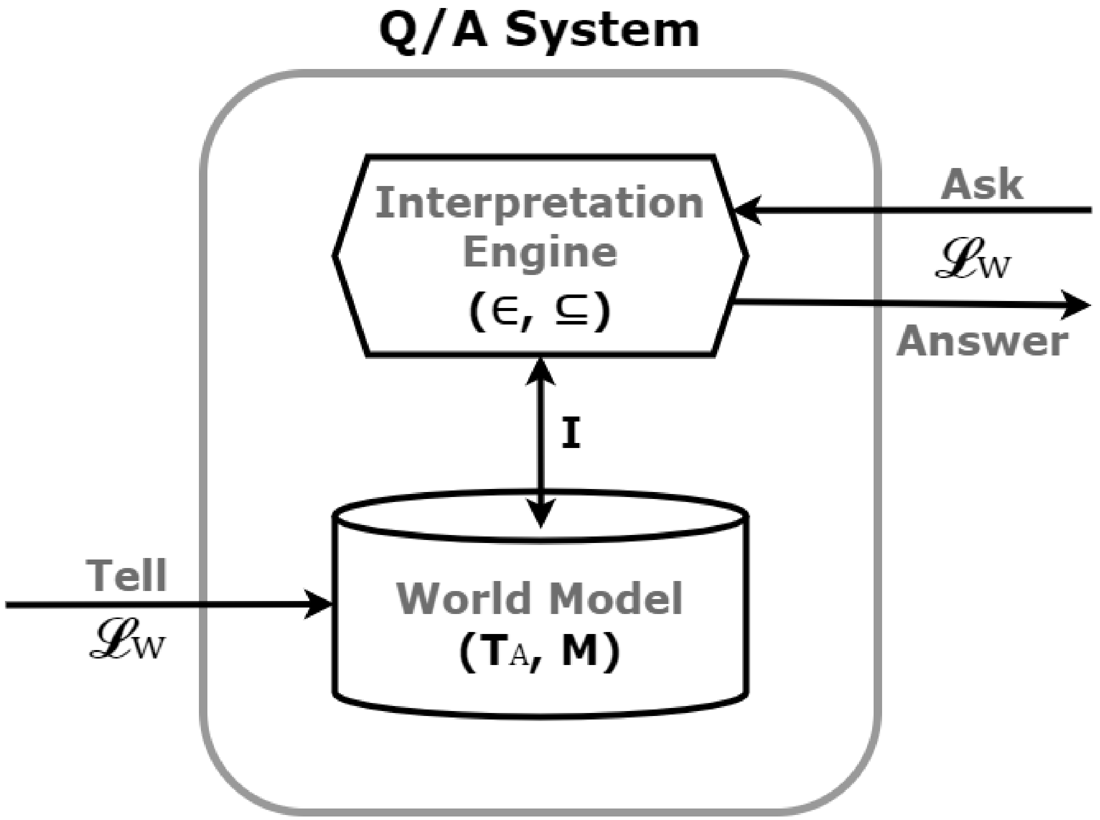
3. Construct $TA = \{a\} \subseteq LA$

4. The model $M = \{f\} \subseteq D$ is automatically defined

NOTE: Agreement is only on linguistic representation, based on a shared understanding of what language means

NOTE 2: agreement at different levels of formality depending on application

Using a world model



Which questions and answers?

Reasoning problems!

Entailment

Definition (Interpretation and entailment) Let W =< LA, D, IA > be a world model. Let T ⊆ LA be a theory and M ∈ D a model of W . Let a ∈ T be an assertion. Then, we write

M |= a to mean I$A(a) \in M$

M |= T to mean I$A(a) \in M$ for all a ∈ T

and say that M **entails** T , or also that M **entails** a.

Reasoning problems (with respect a world model)

Reasoning Problem (Model checking) Given T and M, check whether M |= T.

Reasoning Problem (Satisfiability) Given T , check whether there exists M such that M |= T .

Reasoning Problem (Validity) Given T , check whether for all M, M |= T .

Reasoning Problem 6.4 (Unsatisfiability) Given T , check whether there is no M such that M |= T .

Reasoning problems (with respect a world model) (cont)

Observation (Query answering in DBs) Query answering in DBs is a sophisticated form of model checking / satisfiability.

The contents of the DB are the reference world model, the query is the theory to be model checked, the answer is the set of instantiations which make the input theory correct.

This can be extended to knowledge graphs (both data and knowledge level, e.g. ER/UML like).

World models

Intensional representation

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