UNIVERSITY OF TRENTO - Italy
Dipartimento di Ingegneria e Scienza dell’Informazione

Know dive

Logic
extensional representation

Sept 29,2023

Representation languages

What definitional languages do not represent – judgements (2)

The existence of representation languages is motivated by the fact there is information that cannot be represented in a definitional language. Examples

- *Negative information* (e.g., what I do not perceive or I perceive as false)
- *Partial information* (e.g., what I perceive partially)
- *Consequential information* (e.g., cause effect)
- *Equivalent information* (e.g., bidirectional cause effect)
- *Mutual exclusion* (e.g., one fact excluding the other)
- *Universal / existential statements* (e.g., all swans are white)
- *… and much more* (beliefs, state transitions in time, …) – NOT in this course

Reasoning problems (with respect a world model) - example

Observation (Models in world models are partial) Models say what is the case. But they say nothing about "the rest" (what they do not mention).

Consider the model described by the assertion "my T-shirt is green".

What about the assertion "my paints are grey?"

What about the assertion "my t-shirt is grey?"

There is a fundamental distinction between partial knowledge and negative knowledge, to be captured by logical reasoning.

What assertional languages do not represent – judgements (2)

- **Judgement:** the process of forming an opinion or evaluation by discerning and comparing.
- **Proposition**: a statement or assertion that expresses a judgement
- **Value of propositions**: A proposition is a formula which can be either true or false; it must be one or the other, and it cannot be both
- **Types of propositions:** Atomic (about atomic formulas) and complex (involving multiple atomic formulas)

Complex formulas - example

We can build complex atomic formulas as follows. If $A1$ and $A2$ are formulas, then, $A1$ xor $A2$, is also a formula where Ai can be an atomic as well as a complex formula. The intuition is that $A1$ xor $A2$ contains one and only one fact between the facts denoted by $A1$ and $A2$. Examples:

”Sofia is a woman” ;

”Sofia is a woman” xor ”Sofia is a man”;

”Sofia is a woman” xor ”Paolo is a woman” ;

(”Sofia is a woman” xor ”Paolo is a woman”) xor ”Paolo is a dog” ;

. . . and so on, with indefinitely long complex formulas.

Representation languages – modeling propositions (and definitions)

Definition 8.1 (Representation language, atomic formulas, complex formulas, representation interpretation function)

Let W =< LA, D, IA > be a world model with IA : LA → D. Let La be such that LA ⊆ La and such that there is a **representation interpretation function** I : La → D, with IA ⊆ I (see previous definitions).

Then, a **representation language** L is defined as

$$L = \{w\} = La \cup Lc, \text{ with } La \subset Lc.$$

where:

- w ∈ LA is an **(atomic) assertion**
- w ∈ La is an **atomic (well-formed) formula (complex and atomic assertion)**
- w ∈ Lc is a **complex (well-formed) formula**
- w ∈ L is a **(well-formed) formula**,

La and Lc are the **language of atomic formulas** and **of complex formulas**, respectively

Entailment

Definition (Entailment relation) Let M ⊆ D, T ⊆ L, w ∈. Then |=, to be read "**entails**", is an **entailment relation** defined as

$$|= \subseteq M \times T$$

We also write

$$M \models T \qquad (M \models w)$$

where M |= T stands for M |= w for all w ∈ T . We say that M **entails**, T (w).

Entailment of an atomic formula

Definition (Entailment of an atomic formula, complex and atomic assertion) If w is an atomic formula then we have

$$M \models w \text{ if and only if } I(w) \in M$$

Observation (Entailment of atomic formulas) Entailment of atomic formulas reduces to their interpretation.

Observation (Entailment of complex formulas) Entailment of complex formulas operates in two steps, similarly to how interpretation functions operate on complex atomic formulas. In the first step, it reduces the entailment of a complex formula to that of its component atomic formulas. In the second step it applies the interpretation function o atomic formulas.

Entailment and interpretation (observations)

Observation (Entailment and interpretation – basics).

- Entailment is between models (sets of facts) and theories (sets of formulas)
- Interpretation is between single formulas and single facts
- Entailment takes a model in input and establishes which theories describe that model (which theories are correct for that models)
- Interpretation takes a formula in input and establishes which fact is described by that formula

Entailment and interpretation (observations)

Observation (Entailment relation)

- Interpretation is a function.
- Entailment is a relation.
- Entailment is a many-to-many relation.
- There may be multiple theories that denote a model and, symmetrically, for the same theory there may be multiple models entailed by it (the latter property being the one which makes entailment a relation).

Entailment - example

Consider complex formulas of the form $A1$ xor $A2$, where Ai is any formula. Let us assume that $A1$ and $A2$ are atomic formulas. Then $A1$ xor $A2$ will be denoted by a model M containing the denotation of $A1$ or by one containing the denotation of $A2$. In formulas:

I($A1$) |= $A1$

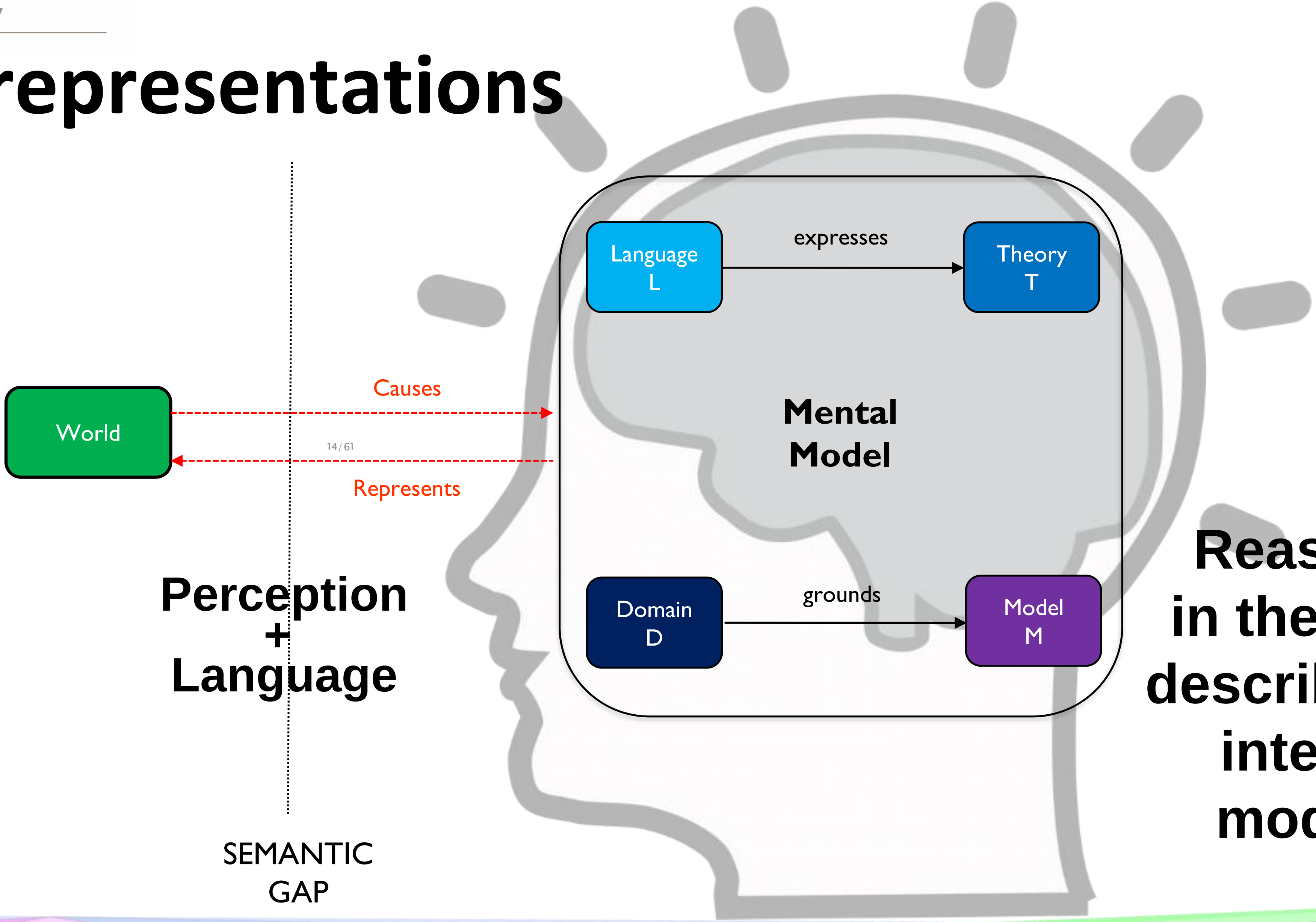
I($A1$) |= $A1$ xor $A2$

I($A1$) not|= $A1$ xor $A2$

{I($A1$), I($A2$)} not|= $A1$ xor $A2$

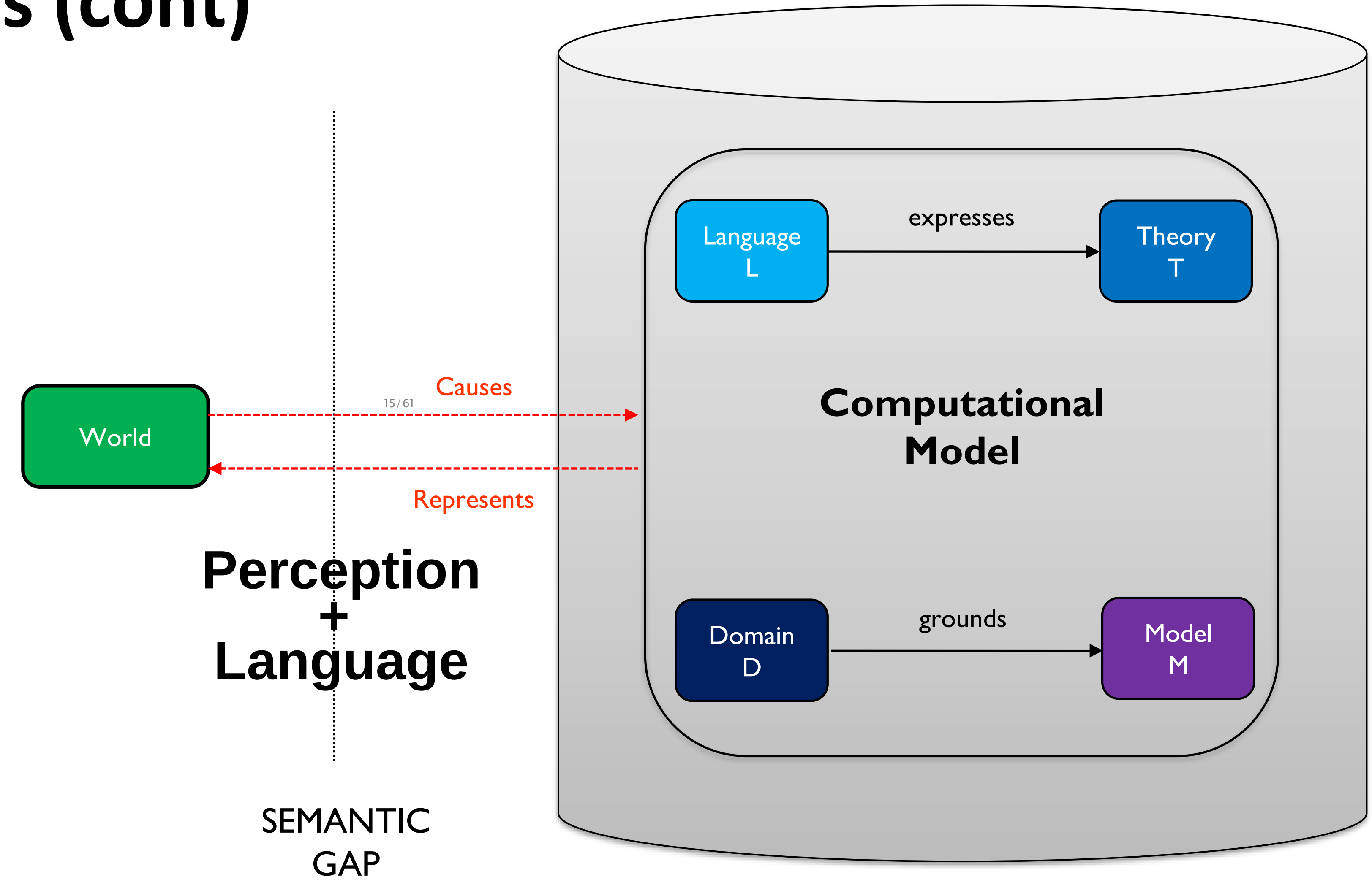
Reasoning as entailment

Mental representations

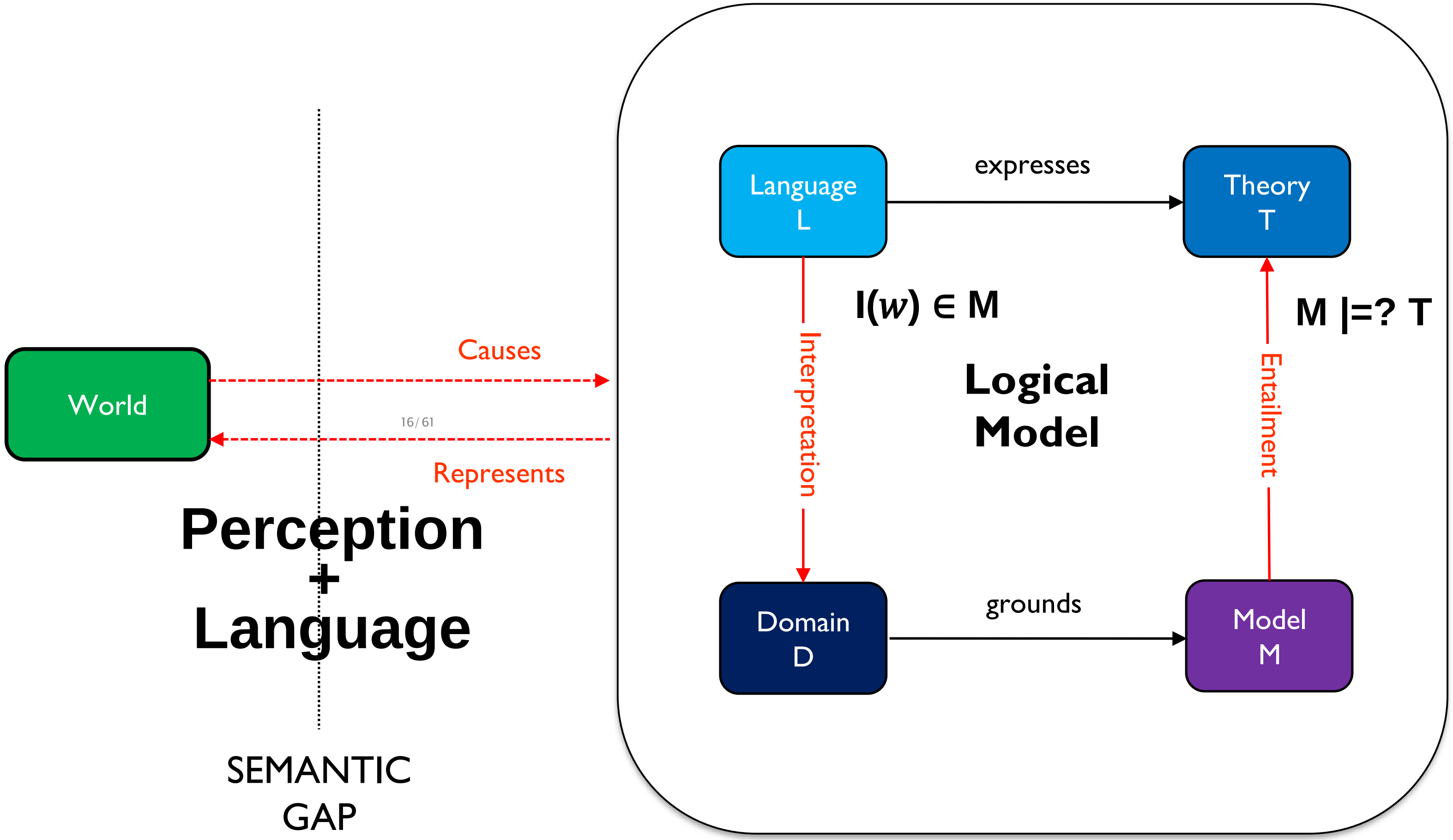


Reasoning in theories T describing the intended models M

Representations (cont)



Reasoning as entailment (cont)



Logical model is a model of both mental and computational model

Reasoning as entailment

Given a Language L and a domain D

Reasoning Problem (Model checking) Given T and M, check whether M |= T.

Reasoning Problem (Satisfiability) Given T , check whether there exists M such that M |= T .

Reasoning Problem (Validity) Given T , check whether for all M, M |= T .

Reasoning Problem 6.4 (Unsatisfiability) Given T , check whether there is no M such that M |= T .

Reasoning as logical entailment

Definition (Logical entailment) Let M ⊆ D be a model and T1, T2 ⊆ L be two theories and w ∈ L a formula. Then we write

T1 |={M} T2 (T1 |={M} w)

and say that T1 **(logically) entails** T2 (w) with respect to the **set of models** {M} if

for all M ∈ {M}, if M |= T1 then M |= T2 (M |= w)

Reasoning as logical entailment properties

Logical entailment - properties

Intuition (Reflexivity)

$$w \models w$$

Observation (Reflexivity) Every fact entails itself. Knowledge asserts itself as being knowledge. This is the essence of what knowledge is about.

Logical entailment – properties (cont.2)

Intuition (Cut)

If $\Gamma \models w1$ and $\Sigma \cup \{w1\} \models w2$ then $\Gamma \cup \Sigma \models w2$

Observation (Cut) There are two ways to interpret cut.

The first and most common is that reasoning can be made efficient by dropping intermediate irrelevant results.

The second is transitivity, namely the fact that reasoning con be composed by chaining independent reasoning sessions, something that people do all the time during their everyday life.

Logical entailment – properties (cont.3)

Intuition (Compactness)

If Γ |= w then there is a finite subset $\Gamma 0 \subseteq \Gamma$ such that $\Gamma 0$ |= w

Observation (Compactness) Consider infinity as the possibility of describing another fact in the process of reasoning. Thus, for instance, natural numbers are infinite and, no matter how many numbers have already been used so far, it is always possible to provide a new one. Compactness says that logical consequence must be computed using a finite set of assumptions. Logical consequence for anhypothetically infinite set of formulas is not a behaviour that is considered of interest.

Logical entailment – properties (cont.4)

Intuition (Monotonicity)

$$\text{If } \Gamma \mid= w \text{ then } \Gamma \cup \Sigma \mid= w$$

Observation (Monotonicity) Monotonicity implements a fundamental and intuitive property of knowledge, for instance of scientific knowledge. If knowledge increases then what can be derived from it via reasoning can only increase. At most it can stay the same if the new piece of knowledge was implied by what is already known.

Logical entailment – properties (cont.5)

Intuition (NonMonotonicity)

$$\Gamma \models w \text{ and } \Gamma \cup \Sigma \ \text{not}\models w$$

Observation (NonMonotonicity) Monotonicity is a property which most often does not hold. This is extensively the case with commonsense reasoning, a topic extensively studied in AI.

How many times getting to know something new has forced us to change our mind? Historical AI example:the belief that all birds fly can be defeated by the fact that penguins are birds and they do not fly.

Historical scientific knowledge example: the discovery that it is the earth rotating around the sun, and not vice versa.

Pratical point of view: the logics used in mathematical reasoning and in formal methods, as applied to, e.g., programming languages, are monotonic, while most logics defined in AI are nonmonotonic. Negation by failure, as implemented in relational DBs is nonmonotonic.

Logics

Logics

Definition (Logic). *L*, defined as

$$\mathcal{L} =< L, D, I, |= >,$$

is a **logic**.

Logics, models and theories

Logics provide the general framework within which logical theories, asserted in representation languages, and models can be defined and compared. Given a logic

$$\mathcal{L} = \langle L, D, I, \models \rangle,$$

we have

$$M = \{f\} \subseteq D$$

$$T = \{w\} \subseteq L$$

Note how the notions of model and domain are the same as with world models

Representation languages - observations

Given a representation language L, a **theory** T is (still) defined as

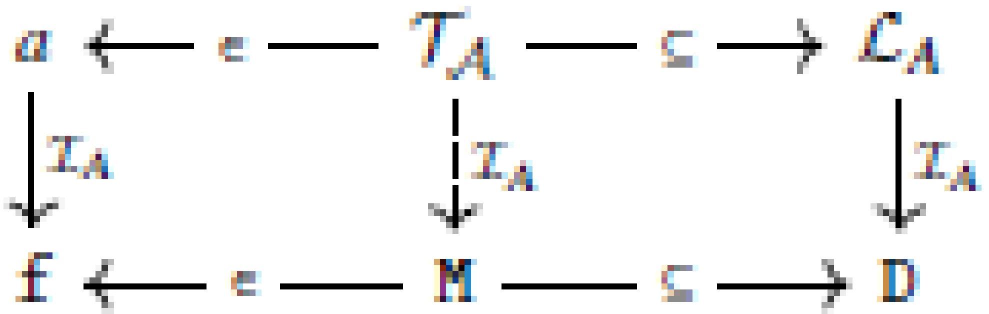
$$T = \{w\} \subseteq L$$

But

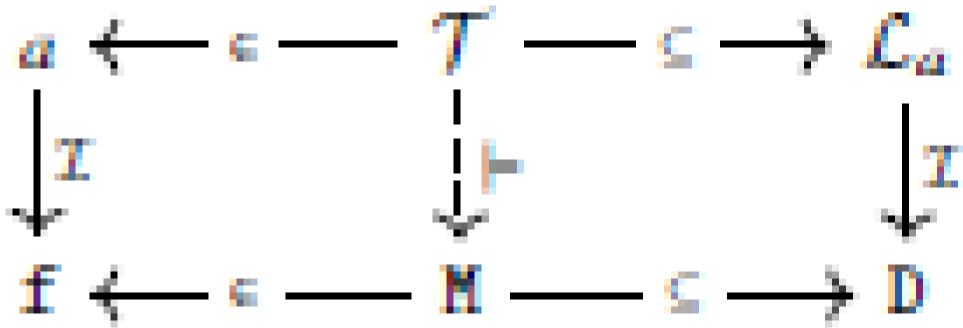
the interpretation function applies ONLY to atomic formulas

World models and Logics- The roles of D, L, I$\mathcal{A}$, M, TA, |=

World models



Logics



Idea: extend world models by logical reasoning

Logics, models and theories – The practice

1. Select a Logic (crucial representation choice)

$$\mathcal{L} = < L, D, I, \models >,$$

2. Agree on L, I (… and therefore D)

3. Agree on |= (… and therefore reasoning principles)

4. Construct TA = $\{a\} \subseteq LA$

5. The model M = {f} ⊆ D is automatically defined

NOTE: Agreement is on linguistic representation, based on a shared understanding of what language means, and on reasoning mechanism (shared understanding?)

NOTE 2: agreement must be formalized

Representation languages - example

The following are examples of representation languages:

- All the natural languages, as used by people in their everyday life;
- Logical languages, that is, subsets of natural languages, with formally defined syntax (language formation rules) and semantics (interpretation function and entailment relation)
- Relational database (DB) languages do not extend to representation languages;
- Entity-relationship (ER) languages do not extend to representation languages
- KGs do not extend to representation languages.

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