Logics

Intensional representation

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Representation language

Definition (Language, intensional representation) Let L = La ∪ Lc be a language. Then its **intensional representation** is

$$Li = < Lia, Lic >$$

where Lia is the **language of atomic formulas, intensionally defined** and Lic is the **language of complex formulas, intensionally defined**.

Language of atomic formulas (reprise)

Definition (Language of atomic formulas, intensional representation) Let $\mathrm{L}i$ =< $\mathrm{L}ia$, $\mathrm{L}ic$ > be a language intensionally defined. Then the **intensional representation**of $\mathrm{L}ia$ is

$$\mathrm{L}ia = < \mathrm{A}a, \{\mathrm{FR}\}a >$$

where: $\mathrm{A}a$ is the **alphabet** of $\mathrm{L}a$ and $\{\mathrm{FR}\}a$ is the set of **formation rules for** $\mathrm{L}ea$ with

$$\mathrm{A}a = < \mathrm{E}, \{\mathrm{C}\}, \{\mathrm{R}\} >$$

$$\mathrm{L}ea = \{w : w \in \mathrm{C}(\{\mathrm{FR}\}a, \mathrm{A}a)\}$$

where: E is a set of **(names of) entities**, {C} is a set of **concepts**, where a conceptis a **name of a class**, {P} and a set of **properties**, where a property is a **name of a relation** and, finally, $\mathrm{C}(\{\mathrm{FR}\}a,\mathrm{A}a)$ is the transitive closure of $\{\mathrm{FR}\}a$ on $\mathrm{A}a$.

Interpretation Function (reprise)

Definition (Interpretation function) Given a language L = La ∪ Lc, La with La = LA ∪ LAC. Let D be a domain. Then an **Interpretation Function** I for La is defined as

$$I : La \rightarrow D \; (I \subseteq La \times D)$$

with

$$I = IA \circ IAC$$

where

$$IAC : La \rightarrow LA \,, \qquad IA : LA \rightarrow D$$

where: IA is an **interpretation function for atomic assertions** and IAC is an **interpretation function for complex assertions**. IA is as defined above.

Furthermore, we say that a fact f ∈ M is the **interpretation** of w ∈ La, and write

$$f = I(a) = a^I$$

to mean that w is a linguistic description of f.We say that f is the **interpretation of** w, or, equivalently, that w **denotes** f.

Language of complex formulas

Definition (Language of complex formulas, formation rules) Let Li =< Lia, Lic > be a language intensionally defined. Then the **intensional representation** of Lic is

$$Lic =< Lea, \{FR\}c >$$

where {FR}c is the set of **formation rules for** Lec with

$$Lec = \{w : w \in C(\{FR\}c, Lea)\}$$

where: Lea is as from above and, C({FR}c, Lea) is the transitive closure of {FR}c on Lea (see above).

Entailment relation

Definition (Entailment relation, intensional representation)
Let M ⊆ D be a model and T ⊆ L a theory. Let the entailment relation be defined as |= ⊆ M × T. Then, the **intensional representation** of |= is

$$|=_i = \langle D, L\{FR\} \rangle$$

where {FR} is the set of **formation rules of** |=e, with

$$|=_e = \{\langle f, w \rangle : f \in M, w \in L, \langle f, w \rangle \in C(\{FR\}, D, L)\}$$

where C({FR},D, L) is the transitive closure of {FR} over <D,L>.

Entailment relation - example

Take L = La ∪ Lc , where the formation rules for Lc, are as follows

<wff> ::= <awff>
<wff> ::= <wff> xor <wff>

Then we define the entailment relation with the following recognition rules:

M |= <awff> ::= I(<awff>)
M |= <wff1> xor <wff2> ::=
M |= <wff1> and M not|= <wff2> |
M not|= <wff1> and M |= <wff2>

We have the following examples (where a, ai are atomic formulas).

M |= a	if I(a) ∈ M
M not|= a	if I(a) ∉ M
M |= $a1$ xor $a2$	if [I($a1$) ∈ M and I($a2$) ∉ M |
	I($a2$) ∈ M and I($a1$) ∉ M]
M |= {$w1$, $w2$}	if M |= $w1$ and M |= $w2$

Logics

Definition (Logic, intensional representation) Let L=< L, D, I, |=>, be a logic defined for the same domain of interpretation Di of ˆW i . Then, the **intensional representation** Li of L is defined as:

Li =< Li , Di , I_i , $|=_i$>, with Li =< Lia, Lic >

and

Lia = < Aa, {FR}a >

Lic = < Lea, {FR}c >

I_i = < Lea, {FR}I >

$|=_i$ = < D,Le, {FR} >

Li , Lia, Lic, Ii , |=i are the **stencils** used to generate a logic

Logics, models and theories – The practice (reprise)

1. Select a Logic – intentional representation (crucial representation choice)

$$\mathit{L} =< L, D, I, |= >,$$

2. Agree on L, I (… and therefore D)

3. Agree on |= (… and therefore reasoning principles)

4. Construct TA = $\{a\} \subseteq$ LA

5. The model M = {f} ⊆ D is automatically defined

NOTE: Agreement is on linguistic representation, based on a shared understanding of what language means, and on reasoning mechanism (shared understanding?)

NOTE 2: agreement must be formalized

Reasoning problems (reprise)

Reasoning Problem (Model checking) Given T and M, check whether M |= T

Reasoning Problem (Satisfiability) Given T , check whether there exists M such that M |= T

Reasoning Problem (Validity) Given T , check whether for all M, M |= T

Reasoning Problem (Unsatisfiability) Given T , check whether there is no M such that M |= T

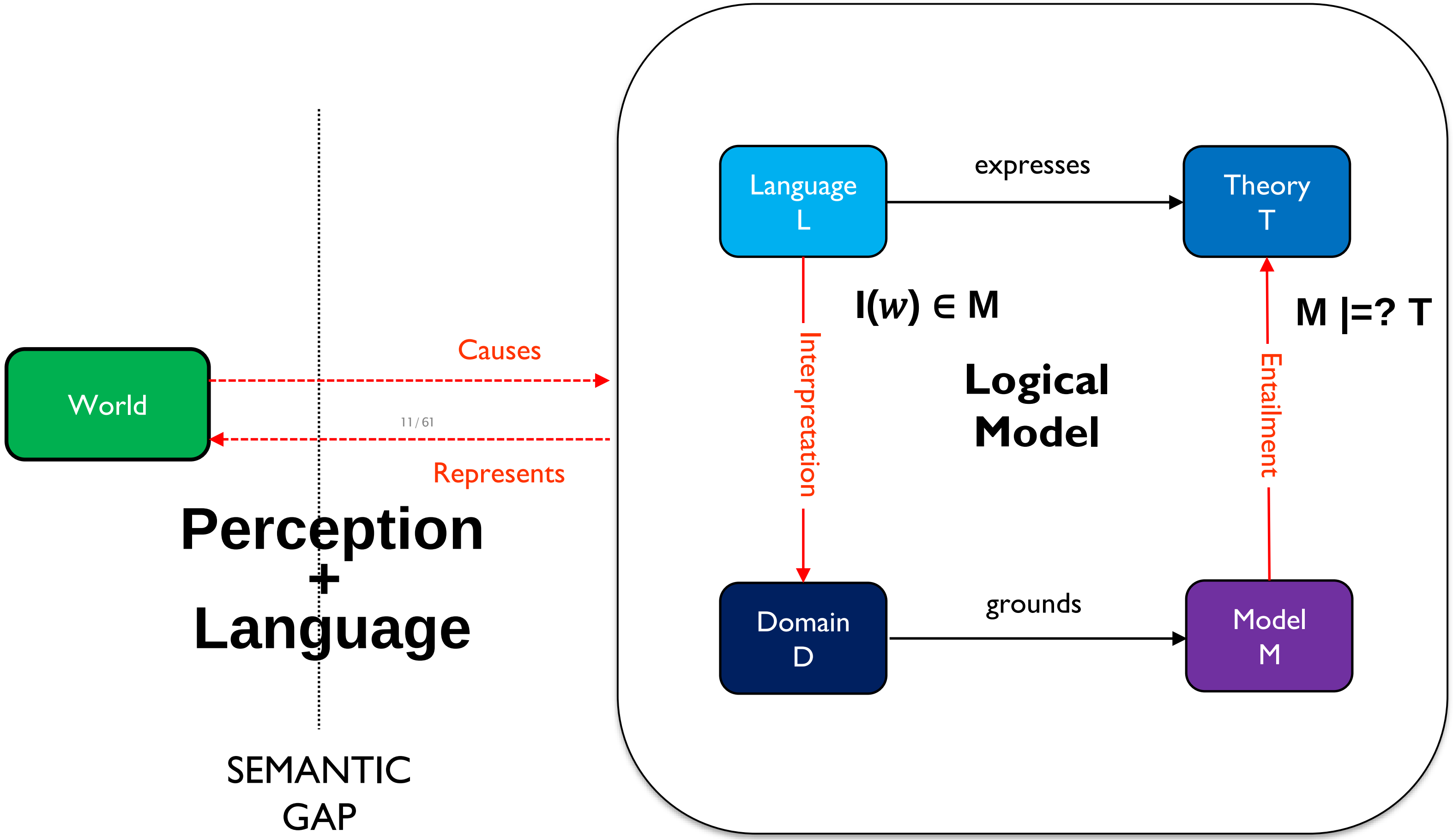
Reasoning Problem (Logical consequence) Given T1, T2 and a set of reference models {M}, check whether

$$T1 \mid=_{\{M\}} T2$$

Reasoning Problem (Logical equivalence) Given T1, T2 and a set of reference models {M}, check whether

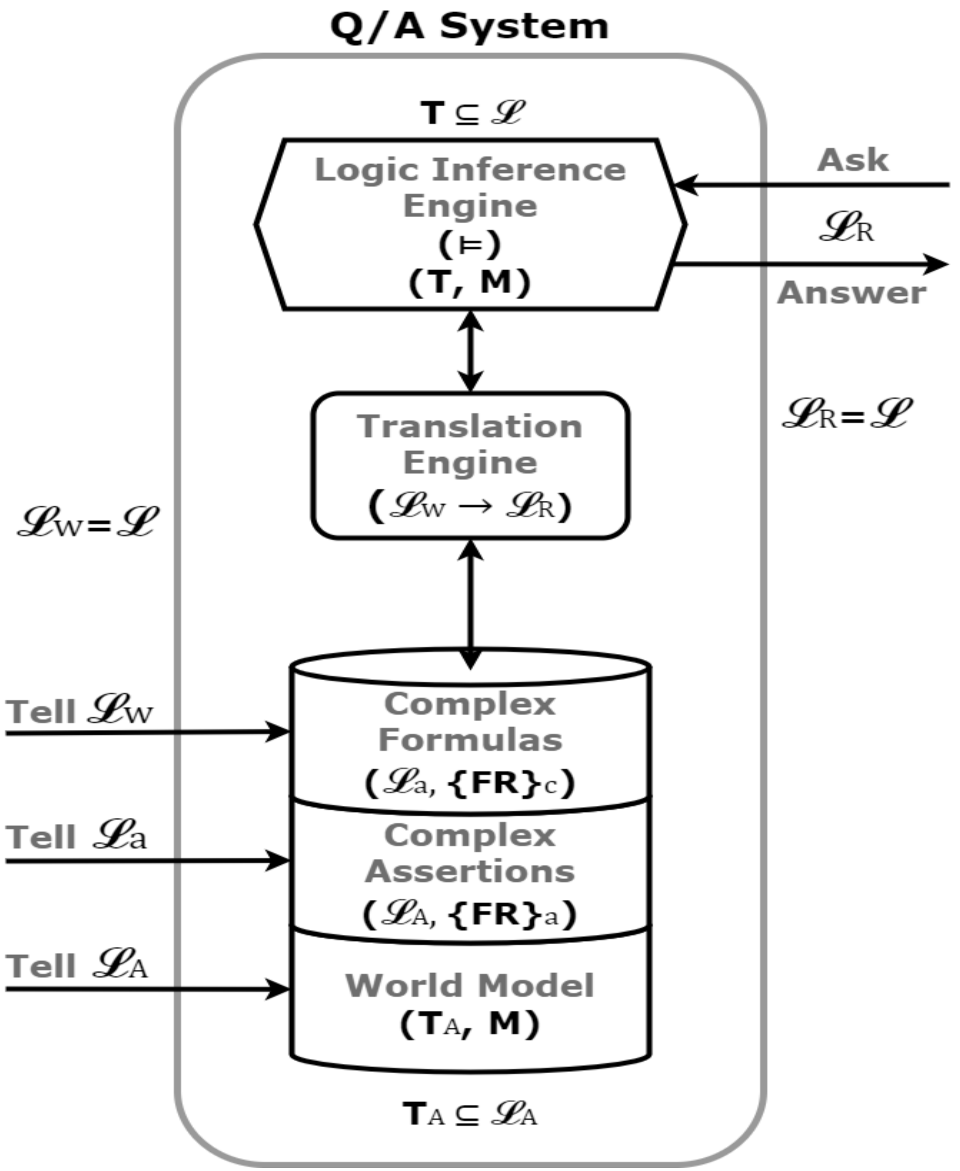
$$T1 \mid=_{\{M\}} T2 \text{ and } T2 \mid=_{\{M\}} T1$$

Reasoning as entailment (cont)



Logical model is a model of both mental and computational model

Using a Logic



Expressivity vs. Efficiency

Observation (Logic, selection trade-offs) Any logics can be characterized by two main parameters:

- **Expressivity**, that is, the level of detail at which the problem is expressed, depending on the syntax of the language of the logic;
- **Computational efficiency**, that is how much it costs, in terms of space and time, to reason and answer queries in that language.

Expressivity vs. Efficiency (cont.)

- More expressivity allows for a more refined and precise modeling of the problem but it also generates longer and more complicated formulas.
- The modeler must find the right trade-off between *expressiveness* and *computational complexity*.
- Here the choice of the representation language L =< La, Lc > is crucial. The computational complexity of both La and Lc ranges in fact from polynomial to exponential and beyond.
- There is also an issue of *(un)decidability*, namely the possibility for the reasoner, on certain queries, to get into an infinite loop, never terminate and, therefore, never return an answer.

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Know dive

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