



UNIVERSITÀ
DI TRENTO



Computational Logic Exercises

Module IV – The Logic of Descriptions (LOD)

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Syntax of LOD

Which of the following symbols are used in LOD ?

$\prod \neg \top \vee \equiv \sqcup \subseteq \rightarrow \leftrightarrow \perp \wedge \vDash$

ANSWER:

$\prod \neg \top \equiv \sqcup \subseteq \perp \vDash$

Syntax of LOD

Which of the following is not a wff in LOD?

1. $\neg \text{MonkeyLow} \sqcup \text{BananaHigh}$
2. $\neg \neg \text{MonkeyLow} \sqcap \text{BananaHigh} \sqsubseteq \neg \text{GetBanana}$
3. $\text{MonkeyLow} \neg \sqcap \text{BananaHigh}$
4. $\text{MonkeyLow} \vee \neg \text{GetBanana}$

ANSWER:

2, 3, 4

Formalization of simple sentences in LOD

The set of games which are not legal	$\text{Game} \sqcap \neg \text{Legal}$
Lakes are locations	$\text{Lake} \sqsubseteq \text{Location}$
Lakes are locations made of water	$\text{Lake} \sqsubseteq \text{Location} \sqcap \exists \text{Madeof.Water}$
Persons can be distinguished into male and female	$\text{Person} \sqsubseteq \text{Male} \sqcup \text{Female}$
Male and Female are disjoint	$\text{Male} \sqsubseteq \neg \text{Female}$
Persons have a birthplace	$\text{Person} \sqsubseteq \exists \text{hasBirthPlace.T}$
The set of documents about “programming in Java” are a subset of the documents about “programming languages” and “computer science”	$\text{JavaProgramming} \sqsubseteq \text{ProgrammingLanguage} \sqcap \text{ComputerScience}$

Formalization of a problem in a LOD theory

Unicorns are mythical horses having a horn.

Unicorn \sqsubseteq mythical \sqcap horse \sqcap \exists has.Horn

There are two kinds of students: master students and PhD students. All PhD students' task is research.

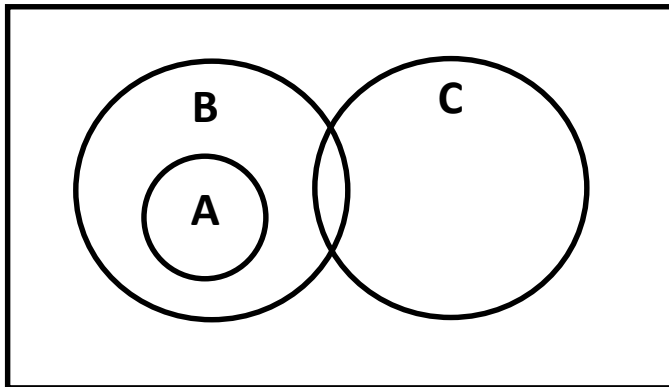
MasterStudent \sqsubseteq Student

PhDStudent \sqsubseteq Student \sqcap \exists hasTask.Research

Venn Diagrams and LOD

Provide a Venn diagram for $A \subseteq B \cap \neg C$

ANSWER:



Define a LOD domain and theory

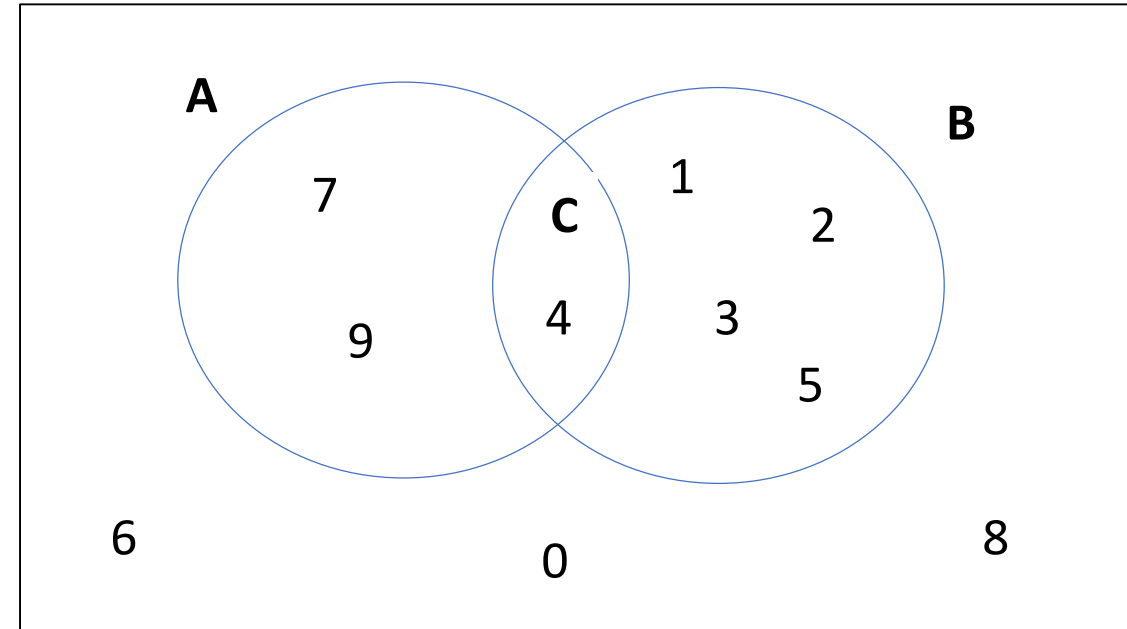
Define a plausible LOD domain D and a theory T for the Venn diagram

ANSWER:

$$D = \langle E, \{C\}, \{P\} \rangle$$

$$E = \{0, \dots, 9\} \quad C = \{A, B, C\} \quad P = \emptyset$$

$$T = \{ C \equiv A \cap B \}$$



Note that: (a) we need to define the language and the interpretation functions; (b) we can define different languages and interpretations

Define a LOD domain and theory

Define a plausible LOD domain D and a theory T for the database table

Employee			
Name	Role	Nationality	Supervises
Fausto	Professor	Italian	Rui
Rui	Student	Chinese	Bisu
Bisu	Student	Indian	-

ANSWER:

$D = \langle E, \{C\}, \{P\} \rangle;$

$E = \{\text{Fausto, Rui, Bisu, Italian, Chinese, Indian}\}$

$C = \{\text{Employee, Professor, Student, Nationality}\}$ $P = \{\text{hasNationality, hasSupervisor}\}$

$T = \{ \text{Professor} \sqsubseteq \text{Employee}; \text{Student} \sqsubseteq \text{Employee};$

$\text{Employee} \sqsubseteq \exists \text{hasNationality.Nationality} \sqcap \exists \text{hasSupervisor.Employee} \}$

Define a LOD domain and theory

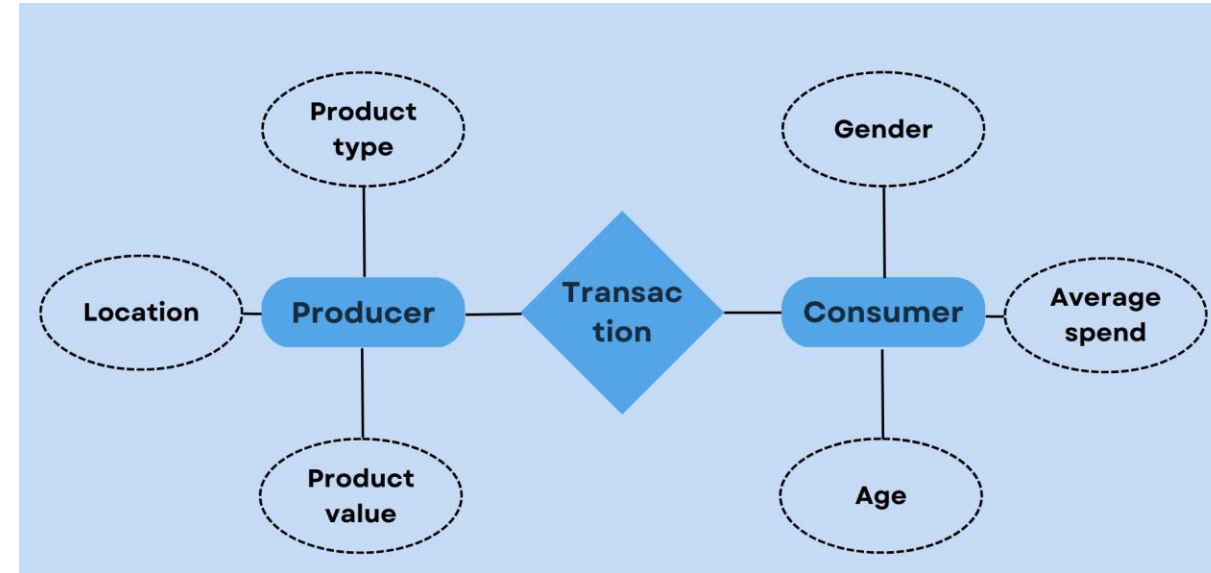
Define a plausible theory T for the ER diagram

ANSWER:

$\text{Producer} \sqsubseteq \text{Entity} \sqcap (\exists \text{ProductType.T} \sqcup \exists \text{Location.T} \sqcup \exists \text{ProductValue.T} \sqcup \exists \text{Transaction.Consumer})$

$\text{Consumer} \sqsubseteq \text{Entity} \sqcap (\exists \text{Gender.T} \sqcup \exists \text{Age.T} \sqcup \exists \text{AverageSpend.T})$

NOTE: in this ER diagram there is no cardinality, but in general we may have it. Consider also the implicit direction of arcs (from left to right).



Define a LOD domain and theory

Define a plausible LOD domain **D**
for the knowledge graph

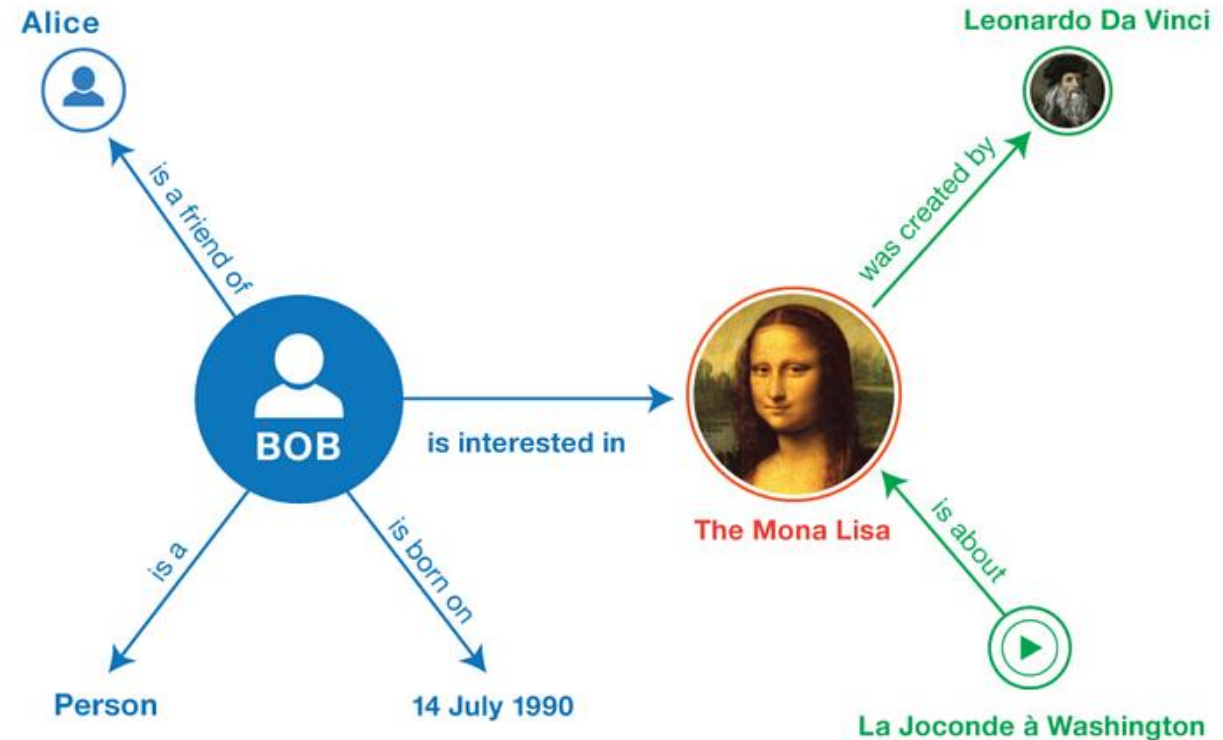
ANSWER (part I):

$D = \langle E, \{C\}, \{P\} \rangle$

$E = \{\text{Alice, Bob, The Mona Lisa, Leonardo Da Vinci, La Joconde à Washington, 14 July 1990}\}$

$C = \{\text{Entity, Person, Picture, File, Date}\}$

$P = \{\text{isFriendOf, interestedIn, isAbout, wasCreatedBy, isBornOn}\}$



Define a LOD domain and theory

Define a plausible LOD theory T for the knowledge graph

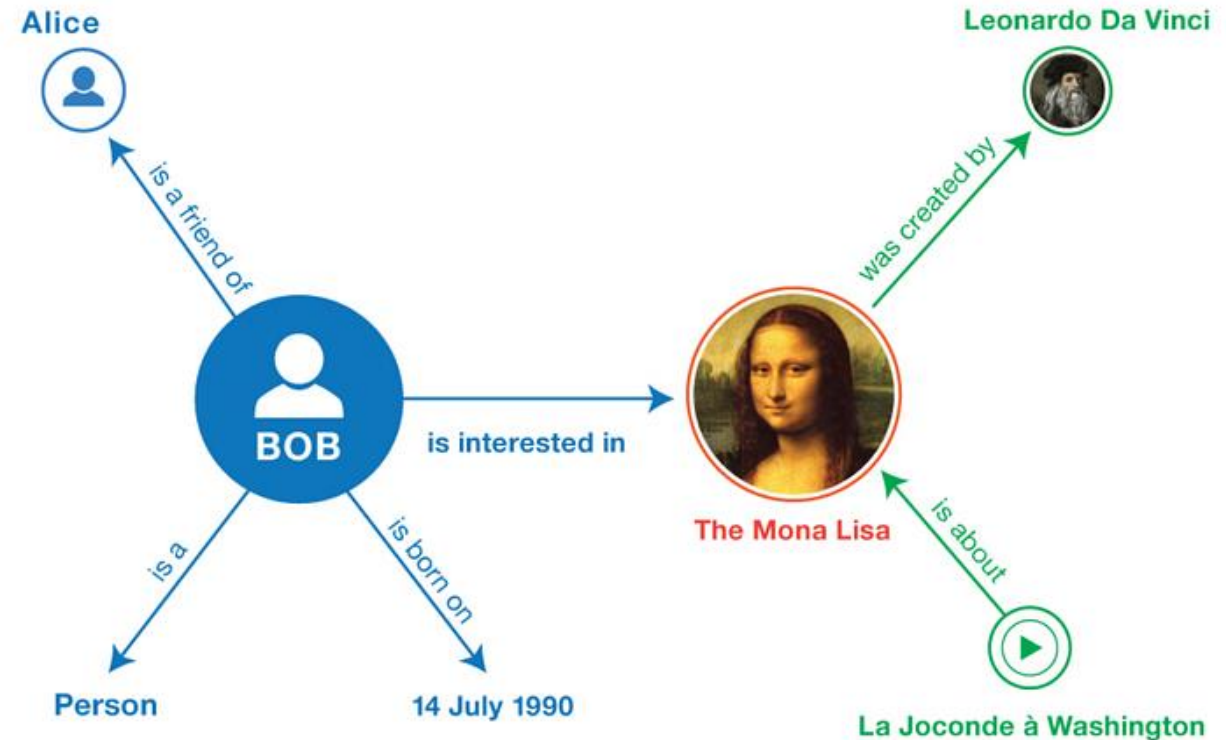
ANSWER (part II):

The theory T can be as follows:

Person \sqsubseteq Entity \sqcap (\exists isFriendOf.Person \sqcup \exists interestedIn.Picture \sqcup \exists isBornOn.Date)

Picture \sqsubseteq Entity \sqcap \exists wasCreatedBy.Person

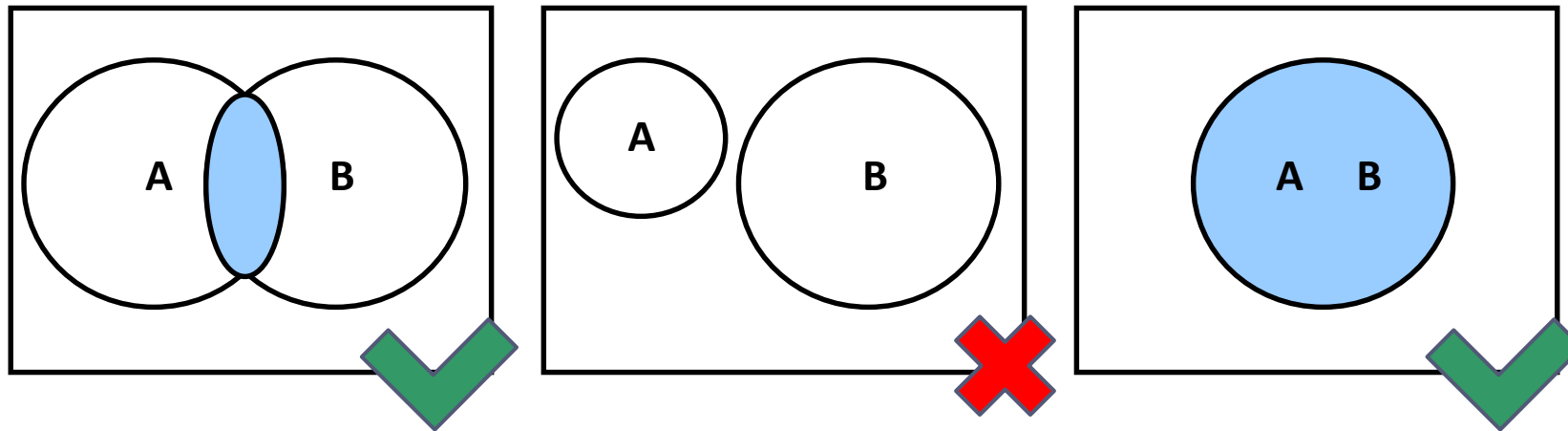
File \sqsubseteq Entity \sqcap \exists isAbout.Picture



Reasoning in LOD

Suppose you have that $M \models A$ and $M \models B$. Does $M \models A \sqcap B$?

ANSWER:

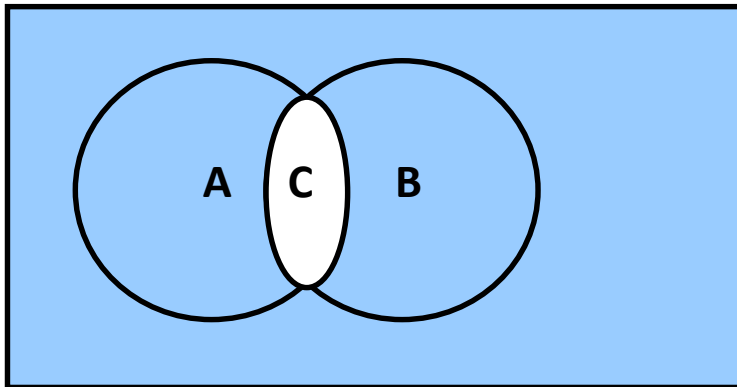


By using Venn Diagrams, we can easily observe that the fact that A and B are not empty does not imply that $A \sqcap B$ is also not empty. Think to the case in which their extensions are disjoint.

Reasoning in LOD

Is the theory $T = \{C \sqsubseteq A, C \sqsubseteq B, \neg(A \sqcap B)\}$ satisfiable?

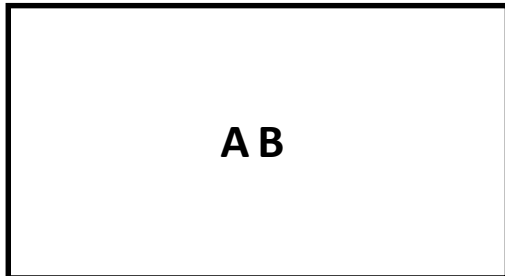
ANSWER: Yes. A case is described below with a Venn Diagram.



Reasoning in LOD

Given the theory $T = \{A \sqsubseteq B, B \sqsubseteq A\}$, does $T \models \neg(A \sqcap B)$?

ANSWER: No. In fact, we can find a counterexample in which $I(A) = I(B)$ but the $I(\neg(A \sqcap B))$ is empty.



Homework

Answer to the following questions

1. What is the purpose of the Logic of Descriptions?
2. What are the key elements of the Logic of Descriptions?
3. What is the form of facts in a domain of the Logic of Descriptions?
4. Do we have negative facts in the Logic of Descriptions?
5. What is the form of assertions in a language of the Logic of Descriptions?
6. What is the form of a theory in the Logic of Descriptions?
7. What is the form of an interpretation function in the Logic of Descriptions?
8. What is entailment in the Logic of Descriptions?
9. What are the reasoning problems in the Logic of Descriptions?

Homework

Modelling exercises

1. Find any linguistic representation of a phenomenon and formalize it in LOD (and/or LOE)
2. Find any Venn Diagram and formalize it in LOD (and/or LOE)
3. Find any ER diagram and formalize it in LOD (and/or LOE)
4. Find any KG and formalize it in LOD (and/or LOE)

Once you define the LOD world model W and the theory T for the problems above, ask yourself:

- a) Can I come up with an equivalent theory, i.e. different world models $W = \langle L, D, I \rangle$ and different assertions in the theory but same intended models?
- b) Is the theory satisfiable? Can I find a model for it?
- c) Can I find logical consequences of the theory?