Chapter 1
$\mathcal{LOD}$ - Logic of Descriptions

1.1 Basic Concepts

Exercise 1.1 (Syntax of $\mathcal{LOD}$) Which of the following symbols are used in LOD ?

$$\sqcap, \neg, \top, \vee, \equiv, \sqcup, \sqsubseteq, \longrightarrow, \longleftrightarrow, \bot, \wedge, \models$$

Exercise 1.2 (Syntax of $\mathcal{LOD}$) Which of the following is not a wff in LOD?

1. $\neg$ MonkeyLow $\sqcup$ BananaHigh
2. $\neg$ $\neg$ MonkeyLow $\sqcap$ BananaHigh $\sqsubseteq$ $\neg$ GetBanana
3. MonkeyLow $\neg$ $\sqcap$ BananaHigh
4. MonkeyLow $\vee$ $\neg$ GetBanana

Exercise 1.3 (TBOX formulas) Which of the following complex formulas are syntactically correct in LOD?

a) A $\equiv$ $\exists$R.C $\sqcap$ $\forall$S.D
b) A $\sqcap$ B $\neg$ $\equiv$ C $\sqcup$ D
c) A $\equiv$ B $\sqcap$ $\neg$C
d) A $\sqsubseteq$ $\neg$C
e) A $\sqsubseteq$ B $\sqcap$ $\exists$R.C
f) A $\sqsubseteq$ B $\sqcap$ $\exists$R.($\forall$S.D)
g) A $\equiv$ B $\sqcup$ $\emptyset$

Exercise 1.4 (Terminology) Is the following TBox a terminology?

- Mother $\equiv$ Woman $\sqcap$ $\exists$hasChild.Person
- Father $\equiv$ Man $\sqcap$ $\exists$hasChild.Person
- Parent $\equiv$ Father $\sqcup$ Mother

Exercise 1.5 (Terminology) Is the following TBox a terminology?

- Mother $\equiv$ Woman $\sqcap$ $\exists$hasChild.Person
- Father $\equiv$ Man $\sqcap$ $\exists$hasChild.Person

- Parent $\sqsubseteq$ Father $\sqcup$ Mother

Exercise 1.6 ($\mathcal{LOD}$ Theory) Answer to the following questions:

1. What is the purpose of the Logic of Descriptions?
2. What are the key elements of the Logic of Descriptions?
3. What is the form of facts in a domain of the Logic of Descriptions?
4. Do we have negative facts in the Logic of Descriptions?
5. What is the form of assertions in a language of the Logic of Descriptions?
6. What is the form of a theory in the Logic of Descriptions?
7. What is the form of an interpretation function in the Logic of Descriptions?
8. What is entailment in the Logic of Descriptions?
9. What are the reasoning problems in the Logic of Descriptions?

Exercise 1.7 (Semantics of $\mathcal{LOD}$) Consider the following statements about the semantics of Logic of Description. Note that the symbol " \" indicates the difference between sets, D is the domain of interpretation, I is the function of interpretation. Indicate which of the following statements about World Logics are TRUE (one or more):

1. $\mathcal{I}(\bot) = \mathcal{I}(\top) \backslash \mathtt{D}$
2. $\mathcal{I}(\exists R.C) = \{d \in \mathtt{D} \mid \text{ exists } e \in \mathtt{D} \text{ with } (d,e) \in \mathcal{I}(R) \text{ and } d \in \mathcal{I}(C)\}$
3. $\mathcal{I}(\forall R.C) = \{d \in \mathtt{D} \mid \text{ forall } e \in \mathtt{D} \text{ if } (d,e) \in \mathcal{I}(R) \text{ then } e \in \mathcal{I}(C)\}$
4. $M \models w_1 \equiv w_2$ if and only if $\mathcal{I}(w_1) \subseteq \mathcal{I}(w_2)$ and $\mathcal{I}(w_2) \subseteq \mathcal{I}(w_1)$
5. $M \models w_1 \perp w_2$ if and only if $\mathcal{I}(w_1) \cap \mathcal{I}(w_2) = \emptyset$

1.2 Translation

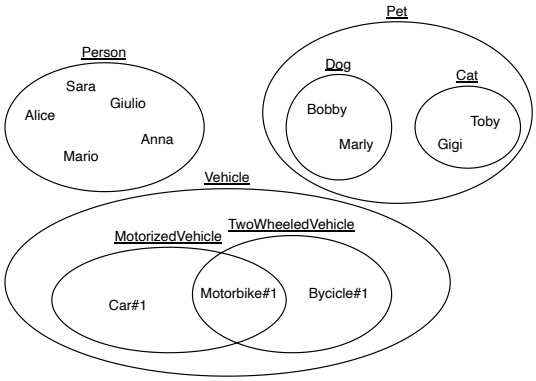
Exercise 1.8 (Informal to Formal with Language of Etype percepts) Formalize the following sentences in natural language using the Language of Etype percepts.

- The set of entities that study in the library.
- The set of books.
- The set of entities that reads books.
- The set of entities that reads only comic books.
- The set of entities that are friends with only entities that study in the Library.

Exercise 1.9 (Informal to Formal with Language of Composite Etype percepts) Formalize the following sentences in natural language using the Language of Composite Etype percepts.

- The set of employees that work at the library.
- The set of black tea and green tea.
- The set of persons that do not drink green tea.
- The set of entities that drink anything but black tea.
- The set of persons that drink green tea but not black tea.

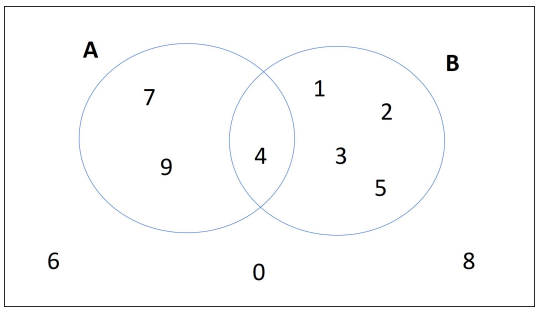
Exercise 1.10 (Interpretation from Intended model) Given the intended model below, determine the set of entities described by the following assertions in Language of Composite Etype percepts.



$$< Alice, Toby >\in ownerOf$$
$$< Giulio, Gigi >\in ownerOf$$
$$< Anna, Marly >\in ownerOf$$
$$< Mario, Motorbike\#1 >\in ownerOf$$
$$< Anna, Bycicle\#1 >\in ownerOf$$

- Person $\sqcap \exists$ownerOf.$\neg$Pet
- $\neg$Vehicle $\sqcap \neg$Dog $\sqcap \forall$ownerOf.$\bot$
- $\exists$ownerOf.(Pet $\sqcap \neg$Cat)
- Person $\sqcap \neg\forall$ownerOf.Cat
- $\neg$(Pet $\sqcup$ Vehicle) $\sqcap$ ($\exists$ownerOf.$\neg$(Pet $\sqcap \neg$Dog) $\sqcap \forall$ownerOf.Cat)
- $\neg$(Pet $\sqcup$ Vehicle) $\sqcup$ ($\exists$ownerOf.$\neg$(Pet $\sqcap \neg$Dog) $\sqcup \forall$ownerOf.Cat)
- $\neg$(Pet $\sqcup$ Vehicle) $\sqcap$ ($\exists$ownerOf.$\neg$(Pet $\sqcap \neg$Dog) $\sqcap \neg\forall$ownerOf.Cat)

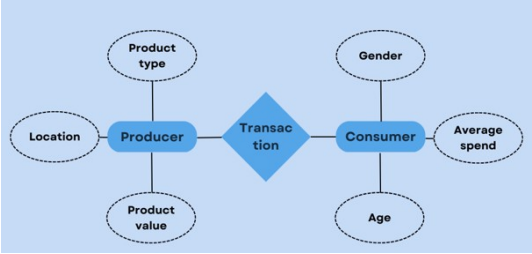
Exercise 1.11 (Define a $\mathcal{LOD}$ domain and theory) Define a plausible $\mathcal{LOD}$ domain $\mathtt{M}$ and a theory $\mathcal{T}$ for the Venn diagram:



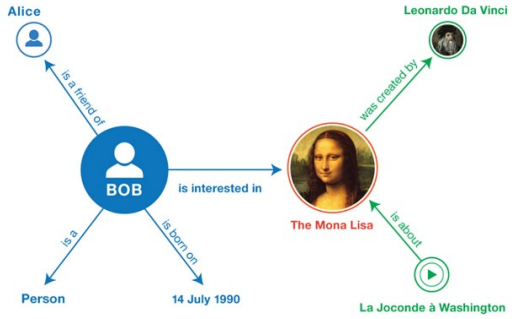
Exercise 1.12 (Define a $\mathcal{LOD}$ domain and theory) Define a plausible $\mathcal{LOD}$ domain $\mathtt{M}$ and a theory $\mathcal{T}$ for the database table:

Employee			
Name	Role	Nationality	Supervises
Fausto	Professor	Italian	Rui
Rui	Student	Chinese	Bisu
Bisu	Student	Indian	-

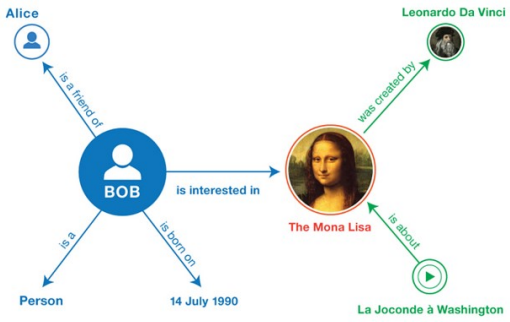
Exercise 1.13 (Define a $\mathcal{LOD}$ domain and theory) Define a plausible $\mathcal{LOD}$ theory $\mathcal{T}$ for the ER:



Exercise 1.14 (Define a $\mathcal{LOD}$ domain and theory) Define a plausible $\mathcal{LOD}$ domain $\mathtt{M}$ for the ER:



Exercise 1.15 (Define a $\mathcal{LOD}$ domain and theory) Define a plausible $\mathcal{LOD}$ theory $\mathcal{T}$ for the ER:



Exercise 1.16 (Translate in $\mathcal{LOD}$) Consider the following concepts: Vehicle, Boat, Bicycle, Car, Device, Wheel, Engine, Axle, Rotation, Water, Human, Driver, Adult, Child.
Formalize in $\mathcal{LOD}$ the following natural language statements:

- Niente
- Tutto
- Humans and vehicles
- Vehicles and not boats
- Wheels or engines and humans
- Adults or children

Exercise 1.17 (Translate in $\mathcal{LOD}$) Consider the following concept and role names:

- Concept names: Vehicle, Boat, Bicycle, Car, Device, Wheel, Engine, Axle, Rotation, Water, Human, Driver, Adult, Child
- Role names: hasPart, poweredBy, capableOf, travelsOn, controls

Formalize in $\mathcal{LOD}$ the following natural language statements:

1. Those vehicles that have wheels and are powered by an engine
2. Those vehicles that have wheels and are powered by a human
3. Those vehicles that travel on water
4. Those objects which have no wheels
5. Those objects which do not travel on water
6. Those devices that have an axle and are capable of rotation
7. Those humans who control a vehicle
8. The drivers of cars

Exercise 1.18 (Translate in $\mathcal{LOD}$) Formalize in $\mathcal{LOD}$ the following natural language statements:

1. Boats have no wheels
2. Cars and bicycles do not travel on water
3. Drivers of cars are adults
4. Humans are not vehicles

5. Wheels or engines are not humans
6. Humans are either adults or children
7. Adults are not children

Exercise 1.19 (Translate in $\mathcal{LOD}$) Formalize in $\mathcal{LOD}$ the following natural language statements:

1. Cars are exactly those vehicles that have wheels and are powered by an engine
2. Bicycles are exactly those vehicles that have wheels and are powered by a human
3. Boats are exactly those vehicles that travel on water
4. Wheels are exactly those devices that have an axle and are capable of rotation
5. Drivers are exactly those humans who control a vehicle

Exercise 1.20 (Translate in $\mathcal{LOD}$)

- The set of games which are not legal
- Lakes are locations
- Lakes are locations made of water
- Persons can be distinguished into male and female
- Male and Female are disjoint
- Persons have a birthplace
- The set of documents about "programming in Java" are a subset of the documents about "programming languages" and "computer science"

Exercise 1.21 (Translate in $\mathcal{LOD}$)

- Unicorns are mythical horses having a horn.
- There are two kinds of students: master students and PhD students. All PhD students' task is research.

Exercise 1.22 (Defining a terminology from natural language definitions) Translate these sentences into a LoD terminology:

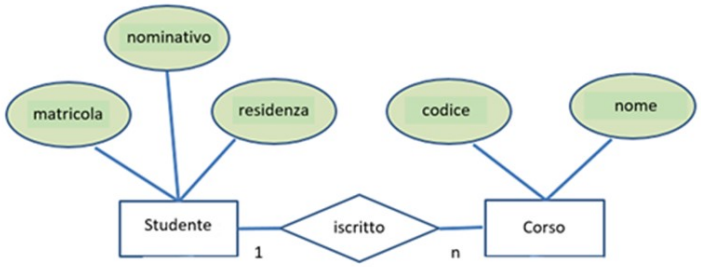
- A lion is a large gregarious predatory feline of Africa or India having a shaggy mane in the male.
- A penguin is a flightless bird of Antarctica having webbed feet.

Exercise 1.23 (Translation of a piece of natural language into $\mathcal{LOD}$) Indicate which of the following statements about the correspondence between natural language and their formalization in the logic of descriptions are TRUE (one or more):

1. The formalization of "Drivers are employees who drive a vehicle" is.
   - Driver $\sqsubseteq$ Employee $\sqcap$ $\exists$driving.Vehicle
2. The formalization of "Drivers are employees who drive an electric vehicle" is.
   - Driver $\sqsubseteq$ Employee $\sqcap$ $\exists$drive.Vehicle $\sqcap$ Electric
3. The formalization of "Drivers are employees who drive an electric vehicle" is.
   - Driver $\sqsubseteq$ Employee $\sqcap$ $\exists$drive.(Vehicle $\sqcap$ Electric)

4. The formalization of ”Drivers are exactly those employees who drive a vehicle and do not drink alcohol” is.
   - Driver ≡ Employee ⊓ ∃drive.Vehicle ⊓ ¬∃drink.Alcohol
5. The formalization of ”Trains are vehicles that do not have wheels” is.
   - Train ⊑ Vehicle ⊓ ¬∀have.Wheel

Exercise 1.24 (From ER to $\mathcal{LOD}$) Let the ER diagram in the figure be given where, following the terminology of ER diagrams, rectangles represent entities (etypes in $\mathcal{LOD}$ terminology), rhombuses represent relationships (Object properties in $\mathcal{LOD}$ terminology), and circles represent attributes, that is, relationships with data types (Data properties in $\mathcal{LOD}$ terminology). It is assumed that the attributes are all mandatory. The notation ”1-n” should be read as the fact that a student can be enrolled in ”n” (more than one) courses. Assume that the diagram to follow is complete in the sense that it represents all and only the entities and relationships of the modeled system.



Assume that you must proceed to formalize this ER model into an etype Graph (ETG), as formalized in the Logic of Descriptions ($\mathcal{LOD}$). Indicate which of the following statements are TRUE (one or more):

1. Student ⊑ ∃enrolled.Course ⊓ ∃residency.⊤
2. Student ⊑ ∀enrolled.Course
3. Course ⊑ ∃enrolled.Student
4. Code ⊥ Matricola

Exercise 1.25 (Define a $\mathcal{LOD}$ domain and theory) Defining a terminology from the following schema:

Thing > Event

Property	Expected Type
Properties from Event	
about	Thing
actor	Person
attendee	Organization or Person

Exercise 1.26 (Venn Diagrams and $\mathcal{LOD}$) Provide a Venn diagram for A $\sqsubseteq$ B $\sqcap$ $\neg$C.

1.3 Reasoning

1.3.1 Entailment

Exercise 1.27 (Reasoning in $\mathcal{LOD}$) Suppose you have that M $\models$ A and M $\models$ B. Does M $\models$ A $\sqcap$ B?

Exercise 1.28 (Reasoning in $\mathcal{LOD}$) Is the theory T= C$\sqsubseteq$A, C$\sqsubseteq$B, $\neg$(A$\sqcap$B) satisfiable?

Exercise 1.29 (Reasoning in $\mathcal{LOD}$) Given the theory T=A$\sqsubseteq$B, B$\sqsubseteq$A, does T $\models$ $\neg$(A$\sqcap$B)?

Exercise 1.30 (Reasoning in $\mathcal{LOD}$) Suppose we describe students in a course as follows:

- Undergraduate $\sqsubseteq$ Student $\sqcap\neg$ Teach
- Bachelor $\equiv$ Student $\sqcap$ Undergraduate
- Master $\equiv$ Student $\sqcap$ $\neg$ Undergraduate
- PhD $\equiv$ Master $\sqcap$ Research
- Assistant $\equiv$ PhD $\sqcap$ Teach

Are all assistants also undergraduates?

Exercise 1.31 (Reasoning in $\mathcal{LOD}$) Suppose we describe students in a course as follows:

- Undergraduate $\sqsubseteq$ Student $\sqcap\neg$ Teach
- Bachelor $\equiv$ Student $\sqcap$ Undergraduate
- Master $\equiv$ Student $\sqcap$ $\neg$ Undergraduate
- PhD $\equiv$ Master $\sqcap$ Research
- Assistant $\equiv$ PhD $\sqcap$ Teach

Are bachelor and master disjoint?

1.3.2 Unfolding

Exercise 1.32 (Unfolding a Concept) Unfold ColouredGuitar:
ElectricGuitar ≡ Guitar ⊓ ∀hasSoundAmplification.withInputJack
ColouredGuitar ≡ ElectricGuitar ⊓ ∃hasColour.String

Exercise 1.33 (Cyclic TBox) Is the following TBox cyclic?

- Woman ≡ Person ⊓ Female
- Man ≡ Person ⊓ ¬Woman
- Mother ≡ Woman ⊓ ∃hasChild.Person
- Father ≡ Man ⊓ ∃hasChild.Person
- Parent ≡ Father ⊔ Mother

Exercise 1.34 (Cyclic TBox) Is the following TBox cyclic?

- Male ≡ ¬Female
- Female ≡ ¬Male

Exercise 1.35 (Logical consequences of a terminology (by unfolding)) Given the this TBOX:

Event ≡ Thing ⊓
∀ about.Thing ⊓ ∃about.Thing ⊓
∀ actor.Person ⊓ ∃actor.Person ⊓
∀attendee.(Person ⊔ Organization) ⊓ ∃attendee.(Person ⊔ Organization)

Which of the following are logical consequences of the TBOX?

- Event ⊑ ∀about.Thing ⊓ ∃about.Thing
- Event ⊑ ∀about.Thing
- Event ⊑ ∃about.Thing
- Event ≡ ∀about.Thing ⊓ ∃about.Thing
- Event ⊑ ∀attendee.Person ⊔ ∀attendee.Organization
- Event ⊑ ∀attendee.Person
- Person ⊑ ¬Organization