Chapter 1
$\mathcal{LOP}$ - Logic of Propositions

1.1 Basic Notions

Exercise 1.1 Which of the following symbols are used in LOP?
$\sqcap \neg \top \vee \equiv \sqcup \sqsubseteq \supset \bot \wedge \models$

Exercise 1.2 Which of the following formulas are NOT syntactically correct in LOP?

1. ¬ MonkeyLow ∨ BananaHigh
2. ¬ ¬ MonkeyLow ¬ BananaHigh
3. MonkeyLow ¬ ¬ BananaHigh
4. MonkeyLow ⊃ ¬ GetBanana
5. MonkeyLow ≡ BananaHigh
6. ¬ (MonkeyLow ∨ BananaHigh) ⊃ BananaHigh

Exercise 1.3 Which of the following are well formed propositional formulas?

1. $\vee pq$
2. $(\neg(p \supset (q \wedge p)))$
3. $(\neg(p \supset (q = p)))$
4. $(\neg(\Diamond(q \vee p)))$
5. $(p \wedge \neg q) \vee (q \supset r)$
6. $p\neg r$

Exercise 1.4 Consider that we have a propositional language L and a theory T written in the language L. Let be I(L) an interpretation of L: the set of atomic propositions and negated atomic propositions that are true. Indicate which of the following statements are true:

1. An interpretation I(L) is such that for every formula A belonging to L we have A belonging to I(L) or ¬ A belonging to I(L).
2. A theory T is such that for every formula A belonging to L we have A belonging to T or ¬ A belonging to T.

3. A model M is an interpretation of a language L that satisfies all the formulas of T.
4. Every model M of a consistent theory T is such that T is a proper or improper subset of M.
5. An inconsistent theory has only one model.
6. Given a language L with N atomic propositions, a tautology (a valid formula) has 2^N models.

Exercise 1.5 Say which of the following statements are true:

1. If T_1 and T_2 are $\mathcal{LOP}$ theories, then, if $T_1 \models A$ and $T_2 \cup \{A\} \models B$ then $T_1 \cup T_2 \models B$
2. T is a theory in $\mathcal{LOP}$ if and only if $T \models A$ implies that $A \in T$
3. A $\mathcal{LOP}$ theory always contains an infinite set of formulas.
4. If the $\mathcal{LOP}$ formula A is satisfiable then $\neg A$ is unsatisfiable
5. If T_1 and T_2 are two $\mathcal{LOP}$ theories then, $T_1 \models A$ and $T_1 \cup T_2 \models A$ if and only if T_2 is not empty

1.2 Informal to formal

Exercise 1.6 Let's consider a propositional language where

- p means *Paola is happy*,
- q means *Paola paints a picture*,
- r means *Renzo is happy*.

Formalize the following sentences:

1. *if Paola is happy and paints a picture then Renzo isn't happy*
2. *if Paola is happy, then she paints a picture*
3. *Paola is happy only if she paints a picture*

Exercise 1.7 Let's consider a propositional language where

- p means *x is a prime number*,
- q means *x is odd*.

Formalize the following sentences:

1. *x being prime is a sufficient condition for x being odd*
2. *x being odd is a necessary condition for x being prime*

Exercise 1.8 Let A =*Aldo is Italian* and B =*Bob is English*. Formalize the following sentences:

1. *Aldo isn't Italian*
2. *Aldo is Italian while Bob is English*
3. *If Aldo is Italian then Bob is not English*
4. *Aldo is Italian or if Aldo isn't Italian then Bob is English*

5. *Either Aldo is Italian and Bob is English, or neither Aldo is Italian nor Bob is English*

Exercise 1.9 Let's consider a propositional language where

- A =*Angelo comes to the party*,
- B =*Bruno comes to the party*,
- C =*Carlo comes to the party*,
- D =*Davide comes to the party*.

Formalize the following sentences:

1. *If Davide comes to the party then Bruno and Carlo come too*
2. *Carlo comes to the party only if Angelo and Bruno do not come*
3. *Davide comes to the party if and only if Carlo comes and Angelo doesn't come*
4. *If Davide comes to the party, then, if Carlo doesn't come then Angelo comes*
5. *Carlo comes to the party provided that Davide doesn't come, but, if Davide comes, then Bruno doesn't come*
6. *A necessary condition for Angelo coming to the party, is that, if Bruno and Carlo aren't coming, Davide comes*
7. *Angelo, Bruno and Carlo come to the party if and only if Davide doesn't come, but, if neither Angelo nor Bruno come, then Davide comes only if Carlo comes*

Exercise 1.10 Angelo, Bruno and Carlo are three students that took the Logic exam. Let's consider a propositional language where

- A =*Aldo passed the exam*,
- B =*Bruno passed the exam*,
- C =*Carlo passed the exam*.

Formalize the following sentences:

1. *Carlo is the only one passing the exam*
2. *Aldo is the only one not passing the exam*
3. *Only one, among Aldo, Bruno and Carlo, passed the exam*
4. *At least one among Aldo, Bruno and Carlo passed*
5. *At least two among Aldo, Bruno and Carlo passed the exam*
6. *At most two among Aldo, Bruno and Carlo passed the exam*
7. *Exactly two, among Aldo, Bruno and Carlo passed the exam*

Exercise 1.11 Let's consider a propositional language where

- A =*Angelo comes to the party*,
- B =*Bruno comes to the party*,
- C =*Carlo comes to the party*,
- D =*Davide comes to the party*.

Formalize the following sentences:

1. *Angelo comes to the party while Bruno doesn't*
2. *Either Carlo comes to the party, or Bruno and Davide don't come*

3. *If Angelo and Bruno come to the party, then Carlo comes provided that Davide doesn't come*
4. *Carlo comes to the party if Bruno and Angelo don't come, or if Davide comes*
5. *If Angelo comes to the party then Bruno or Carlo come too, but if Angelo doesn't come to the party, then Carlo and Davide come*

Exercise 1.12 Let's consider a propositional language where p stands for ''Paola is happy'', q stands for ''Paola points a picture'', and r stands for ''Renzo is happy''. Formalize the following sentences:

1. ''if Paola is happy and points a picture then Renzo isn't happy''
2. ''if Paola is happy, then she points a picture''
3. ''It is not the case that Paola is happy and she does not point a picture''

Exercise 1.13 Translate natural language sentences into LOP:

- David or Bruno come to the party
- Bruno will come to the party
- Bruno will come to the party, unless David is there
- Carlo comes to the party, therefore David comes too
- Either David or Bruno come to the party
- Neither Carlo nor David will come to the party

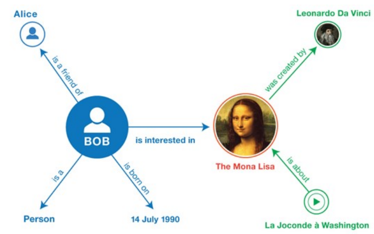
Exercise 1.14 Translate natural language sentences into LOP:

- If David comes to the party then Bruno and Carlo come too
- Angelo will come to the party, provided that Bruno comes and Carlo does not
- Carlo comes to the party given that David doesn't come, but, if David comes, then Bruno doesn't come
- Carlo comes to the party only in case Angelo and Bruno do not come
- A sufficient condition for Angelo coming to the party, is that Bruno and Carlo aren't coming
- A necessary and sufficient condition for Angelo coming to the party, is that Bruno and Carlo aren't coming

Exercise 1.15 [Modelling: Define the theory for a certain problem] Passing the exam. If you play and you study you'll pass the exams, while if you play and don't study you won't pass. Thus, if you play, then you study and you'll pass the exams, or you don't study and you won't pass it.

Exercise 1.16 [Modelling: Define the theory for a certain problem] The effect of bananas. Bananas may differ in many ways. However, there are red and yellow bananas. I like bananas, but I eat only yellow bananas. If I do not eat at least a banana I get crazy.

Exercise 1.17 (From a knowledge graph to a $\mathcal{LOP}$ theory) Define a $\mathcal{LOP}$ from this $\mathcal{KG}$:



Exercise 1.18 (From informal to formal) Indicate which single logical connective should be used in the translation into Logical Propositions ($\mathcal{LOP}$) of the sentence ”I am going to Rome by train or plane.”

1. $\wedge$ (and)
2. $\vee$ (or)
3. $\neg$ (not)
4. $+$ (xor)
5. $\supset$ (implication)
6. $\equiv$ (equivalence)
7. No logical connective

Exercise 1.19 (From informal to formal) Indicate which single logical connective should be used in the translation into Logical Propositions ($\mathcal{LOP}$) of the sentence ”I took the umbrella but got wet.”

1. $\wedge$ (and)
2. $\vee$ (or)
3. $\neg$ (not)
4. $+$ (xor)
5. $\supset$ (implication)
6. $\equiv$ (equivalence)
7. No logical connective

Exercise 1.20 (From informal to formal) Indicate which single logical connective should be used in the translation into Logical Propositions ($\mathcal{LOP}$) of the sentence ”Fausto’s house and Vincenzo’s house are close together.”

1. $\wedge$ (and)
2. $\vee$ (or)
3. $\neg$ (not)
4. $+$ (xor)
5. $\supset$ (implication)
6. $\equiv$ (equivalence)
7. No logical connective

1.3 Reasoning problems

Exercise 1.21 Let's consider the interpretation v where $v(p) = \mathrm{F}$, $v(q) = \mathrm{T}$, $v(r) = \mathrm{T}$. Does v satisfy the following propositional formulas?

1. $(p \supset \neg q) \vee \neg(r \wedge q)$
2. $(\neg p \vee \neg q) \supset (p \vee \neg r)$
3. $\neg(\neg p \supset \neg q) \wedge r$
4. $\neg(\neg p \supset q \wedge \neg r)$

Exercise 1.22 Socrate says: *If I'm guilty, I must be punished; I'm guilty. Thus I must be punished.* Is the argument logically correct?

Exercise 1.23 Socrate says: *If I'm guilty, I must be punished; I'm not guilty. Thus I must not be punished.* Is the argument logically correct?

Exercise 1.24 Socrate says: *If I'm guilty, I must be punished; I must not be punished. Thus I'm not guilty.* Is the argument logically correct?

Exercise 1.25 Socrate says: *If I'm guilty, I must be punished; I must be punished. Thus I'm guilty.* Is the argument logically correct?

Exercise 1.26 Formalize the following arguments and verify whether they are correct:

- *If Carlo won the competition, then either Mario came second or Sergio came third. Sergio didn't come third. Thus, if Mario didn't come second, then Carlo didn't win the competition.*
- *If Carlo won the competition, then either Mario came second or Sergio came third. Mario didn't come second. Thus, if Carlo won the competition, then Sergio didn't come third.*
- *If Carlo won the competition, then Mario came second and Sergio came third. Mario didn't come second. Thus Carlo didn't win the competition.*
- *If Carlo won the competition, then, if Mario came second then Sergio came third. Mario didn't come second. Thus, either Carlo won or Sergio arrived third*
- *If you play and you study you'll pass the exams, while if you play and don't study you won't pass. Thus, if you play, either you study and you'll pass the exams, or you don't study and you won't pass.*

Exercise 1.27 Compute the truth table of $(F \vee G) \wedge \neg(F \wedge G)$.

Exercise 1.28 Use the truth tables method to determine whether $(p \supset q) \vee (p \supset \neg q)$ is valid.

Exercise 1.29 Use the truth tables method to determine whether $(\neg p \vee q) \wedge (q \supset \neg r \wedge \neg p) \wedge (p \vee r)$ (denoted with φ) is satisfiable.

Exercise 1.30 Use the truth tables method to determine whether the formula $\varphi : p \wedge \neg q \supset p \wedge q$ is a logical consequence of the formula $\psi : \neg p$.

Exercise 1.31 Use the truth tables method to determine whether $p \supset (q \wedge \neg q)$ and $\neg p$ are logically equivalent.

Exercise 1.32 Compute the truth tables for the following propositional formulas:

- $(p \supset p) \supset p$
- $p \supset (p \supset p)$
- $p \vee q \supset p \wedge q$
- $p \vee (q \wedge r) \supset (p \wedge r) \vee q$
- $p \supset (q \supset p)$
- $(p \wedge \neg q) \vee \neg(p \leftrightarrow q)$

Exercise 1.33 Use the truth table method to verify whether the following formulas are valid, satisfiable or unsatisfiable:

- $(p \supset q) \wedge \neg q \supset \neg p$
- $(p \supset q) \supset (p \supset \neg q)$
- $(p \vee q \supset r) \vee p \vee q$
- $(p \vee q) \wedge (p \supset r \wedge q) \wedge (q \supset \neg r \wedge p)$
- $(p \supset (q \supset r)) \supset ((p \supset q) \supset (p \supset r))$
- $(p \vee q) \wedge (\neg q \wedge \neg p)$
- $(\neg p \supset q) \vee ((p \wedge \neg r) \leftrightarrow q)$
- $(p \supset q) \wedge (p \supset \neg q)$
- $(p \supset (q \vee r)) \vee (r \supset \neg p)$

Exercise 1.34 Use the truth table method to verify whether the following logical consequences and equivalences are correct:

- $(p \supset q) \models \neg p \supset \neg q$
- $(p \supset q) \wedge \neg q \models \neg p$
- $p \supset q \wedge r \models (p \supset q) \supset r$
- $p \vee (\neg q \wedge r) \models q \vee \neg r \supset p$
- $\neg(p \wedge q) \equiv \neg p \vee \neg q$
- $(p \vee q) \wedge (\neg p \supset \neg q) \equiv q$
- $(p \wedge q) \vee r \equiv (p \supset \neg q) \supset r$
- $(p \vee q) \wedge (\neg p \supset \neg q) \equiv p$
- $((p \supset q) \supset q) \supset q \equiv p \supset q$

Exercise 1.35 Suppose we know that:

- *if Paolo is thin, then Carlo is not blonde or Roberta is not tall*
- *if Roberta is tall then Sandra is lovely*
- *if Sandra is lovely and Carlo is blonde then Paolo is thin*
- *Carlo is blonde*

Can we deduce that *Roberta is not tall* ?

Exercise 1.36 Anna and Barbara carpool to work. On any day, either Anna drives Barbara or Barbara drives Anna. In the former case, Anna is the driver and Barbara is the passenger; in the latter case Barbara is the driver and Anna is the passenger. Formalize the problem using the following propositions:

1. *Anna drives Barbara*
2. *Barbara drives Anna*
3. *Anna is the driver*
4. *Barbara is the driver*
5. *Anna is the passenger*
6. *Barbara is the passenger*

Exercise 1.37 Check if $(P \wedge Q) \vee (R \supset S)$ is satisfiable.

Exercise 1.38 Check if $(P \wedge Q) \vee (R \supset S)$ is valid.

Exercise 1.39 Use the truth table method to determine whether $\neg p$ is a logical consequence of the formula A = $\{q \vee r \wedge q \supset \neg p \wedge \neg(r \wedge p)\}$

Exercise 1.40 Use the truth tables method to determine whether $p \supset (q \wedge \neg q)$ and $\neg p$ are logically equivalent.

Exercise 1.41 Let A be an atomic proposition in $\mathcal{LOP}$ and B a generic propositional variable. Indicate which of the following answers are correct:

1. ¬ A it is unsatisfiable
2. ¬ A ∨ B is satisfiable
3. (A ∧ B) ⊃ B is satisfiable
4. (A ∧ B) ⊃ is valid
5. (¬ A ∨ A) is satisfiable, but not valid
6. If B is a tautology, then A ⊃ B is valid

Exercise 1.42 Find an interpretation that does not satisfy the following formula:

$$(\neg(A \vee B) \wedge (C \wedge \neg D)) \supset (\neg C \vee B)$$

specifying for each propositional variable the corresponding truth value assigned by the interpretation (True, False, Unassigned)

Exercise 1.43 (Truth tables) Given the propositions A and B, calculate the Truth Table of the following formulas:

- A ∧ B
- A ∨ B
- A ≡ B

Exercise 1.44 (Finding models of formulas using Truth Tables) List the models for the following formulas, given the propositions A, B, C:

- A ∧ ¬B
- (A ∧ B) ∨ (B ∧ C)

- $(A \vee B) \supset C$
- $\neg A \equiv B \equiv C$

Exercise 1.45 (Finding models of formulas using Truth Tables) List the models for the formula P: $(A \vee B) \supset \neg C$.

Exercise 1.46 (Reasoning with Truth Tables) Prove that the formula P is unsatistiable, where P: $(A \wedge B) \wedge \neg B$. RECALL: a formula is unsatistiable if it is false for all assignments (there are no models).

Exercise 1.47 (Reasoning with Truth Tables) Prove that the formula P is valid, where P: $\neg(A \supset B) \supset (A \wedge \neg B)$. RECALL: a formula is valid if it is true for all assignments (all assignments are models for it)

Exercise 1.48 (Modeling and solving a problem with LOP) A very special island is inhabited only by knights and knaves. Knights always tell the truth, and knaves always lie. You meet two inhabitants: Zoey and Mel.

- Zoey tells you that Mel is a knave.
- Mel says 'Neither Zoey nor I are knaves'.

Can you determine what are they? (who is a knight and who is a knave?)

Exercise 1.49 (Prove equivalence) Let A, B, C, D, E be propositions. Prove by using basic facts that: $((A \vee \neg E) \wedge (\neg B \vee \neg D)) \supset B \equiv (\neg A \wedge E) \vee ((B \wedge D) \vee B)$. Then try yourself also for:

- $\neg(A \vee (A \supset B)) \equiv \neg A \wedge \neg B$
- $\neg(A \supset B) \wedge (\neg B \wedge \neg C) \equiv A \wedge \neg(B \vee C)$
- $\neg(A \supset \neg B) \wedge (\neg B \wedge \neg C) \equiv \bot$
- $(C \supset (A \supset \neg B) \supset (C \supset \neg A)$

Exercise 1.50 (Prove entailment) RECALL: $\Gamma \models \Phi$ iff all models satisfying the formulas in Γ also satisfy Φ.
Let A, B, C be propositions. If $A \models B \wedge C$, then $A \models B$ or $A \models C$ or both?

Exercise 1.51 (Prove entailment) RECALL: If for all and only the atomic propositions P occurring in a formula A we have I(P) = I'(P), then $I \models A$ iff $I' \models A$. That is:

- The truth value of atomic propositions which occur in A fully determines the truth value of A
- The truth value of the atomic propositions which do not occur in A play no role in the computation of the truth value of A;

EXERCISE: Let X, Y, Z be atomic propositions. Let us take $A = (X \wedge \neg Z)$ and interpretations I = X, Y and I' = X Then $I \models A$ iff $I' \models A$.

Exercise 1.52 (Reasoning by applying entailment properties) Let A and B be propositions. Prove that the formula P is valid, where P: $(A \supset B) \supset (\neg A \vee B)$

Exercise 1.53 (Prove entailment) Let A, B, C, D, E be propositions, and given that:

- P = (A ∨ B) ∧ (¬C ∨ ¬D ∨ E)
- Q1 = A ∨ B
- Q2 = (A ∨ B ∨ C) ∧ ((B ∧ C ∧ D) ⊃ E)
- Q3 = (A ∨ B) ∧ (¬D ∨ E)

Does P ⊨ Qi ?

Exercise 1.54 (Prove entailment using Truth Tables) Let A, B, C, D, E be propositions, and given that:

- P = (A ∨ B) ∧ (¬C ∨ ¬D ∨ E)
- Q1 = A ∨ B
- Q2 = (A ∨ B ∨ C) ∧ ((B ∧ C ∧ D) ⊃ E)
- Q3 = (A ∨ B) ∧ (¬D ∨ E)

(1) List all truth assignments such that P ⊨ Qi
(2) Is there any assignment such that P ⊨ Qi for all i?

Exercise 1.55 (Models and theories) Let A and B be propositions. Let T1= (A ⊃ B) and T2= ¬A ∧ B. Say which of the following sentences is true.

- T1 has 2 models
- T2 has 1 model
- T1 and T2 have 2 models in common
- Does T1 ⊨ T2?
- Does T2 ⊨ T1?
- T1 and T2 are maximal theories for M = B

	A	B	A ⊃ B	¬A ∧ B
M1	T	T	T	F
M2	T	F	F	F
M3	F	T	T	T
M4	F	F	T	F

Exercise 1.56 (Models and theories) Let A and B be propositions. Let T1= (A ≡ B) and T2= ¬A, B. Say which of the following sentences is true.

- T1 has 2 models
- T2 has 1 model
- T1 and T2 have 1 model in common
- Does T1 ⊨ T2?
- Does T2 ⊨ T1?
- T2 is a maximal theory for M = B
- T1 is a maximal theory for M = A, B
- The minimal model of T2 is M = B
- The minimal model of T1 is M = A, B

	A	B	A ≡ B	¬A
M1	T	T	T	F
M2	T	F	F	F
M3	F	T	F	T
M4	F	F	T	T

Exercise 1.57 (Models and theories) RECALL: A minimal Model M of T is the intersection of all models of T.
Let A, B, C be propositions. Given the theory T = ¬A ∨ ¬B, C ∧ ¬B, say which of the following sentences is true.

- T has 2 models
- T has 1 model
- There is no minimal model of T
- T satisfies M = A
- The minimal model is M = C
- The minimal model is M = A, C

A	B	C	¬A ∨ ¬ B	C ∧ ¬ B
T	T	T	F	F
T	F	T	T	T
F	T	T	T	F
F	F	T	T	T
T	T	F	F	F
T	F	F	T	F
F	T	F	T	F
F	F	F	T	F

Exercise 1.58 (Models and theories) RECALL: A minimal Model M of T is the intersection of all models of T.
Let A, B, C be propositions. Given the theory T = A ∧ ¬B, A ∧ C, C ∧ ¬B, say which of the following sentences is true.

- T has 2 models
- T has 1 model
- There is no minimal model of T
- T satisfies M = A
-
- The minimal model is M = A
- The minimal model is M = A, C

A	B	C	A ∧ ¬ B	A ∧ C	C ∧ ¬ B
T	T	T	F	T	F
T	F	T	T	T	T
F	T	T	F	F	F
F	F	T	F	F	T
T	T	F	F	F	F
T	F	F	T	F	F
F	T	F	F	F	F
F	F	F	F	F	F

Exercise 1.59 Say which of the following statements are true:

1. If a fact is an element of a $\mathcal{LODE}$ domain then the proposition representing it in the $\mathcal{LOP}$ language describing the domain is true
2. If a proposition in a $\mathcal{LOP}$ language is true then the fact represented in the $\mathcal{LODE}$ domain described by the $\mathcal{LOP}$ language is true
3. Given an $\mathcal{EG}$ formalized in a $\mathcal{LODE}$ logic, for each assertion in the $\mathcal{LODE}$ language there can exist multiple (atomic) propositions in the corresponding theory formalized in $\mathcal{LOP}$
4. An Entity Graph (ABOX) formalized in $\mathcal{LODE}$ logic can have multiple models.

Exercise 1.60 (Basic Facts Entailment) Using the properties of the logic of propositions in $\mathcal{LOP}$ language indicate whether:

$$((p \vee s) \supset \neg q) \supset r \equiv (p \wedge r) \vee (s \wedge r) \vee (q \wedge r)$$

NOTE: It is suggested that the left formula be rewritten, using the properties of logic, until the right formula is arrived at.

Exercise 1.61 ($\mathcal{LOP}$ Models and Theories) Given propositions X, Y and Z and two theories T1 = $\{\neg(X \equiv Y), Y \wedge Z\}$ and T2 = $\{\neg X, Y, Z \supset Y\}$ in $\mathcal{LOP}$ language, indicate which of the following statements are true (one or more).

A) T1 has 2 models
B) T2 has 2 models
C) T1 and T2 have 1 model in common
D) T1 $\models$ T2
E) T2 $\models$ T1
F) M = {Y, Z} is a model for T1
G) M = {X, Y} is a model for T2
H) The minimal model (”minimal model”) of T2 exists and is M = {Y}