

# Chapter 1

## LOI

### 1.1 Basic Concepts

**Exercise 1.1** Non Logical symbols:

- constants  $a, b$ ;
- functions  $f_1, g_2$ ;
- predicates  $p_1, r_2, q_3$ ;

Say whether the following strings of symbols are well-formed FOL formulas or terms:

1.  $q(a)$
2.  $p(y)$
3.  $p(g(b))$
4.  $\neg r(x, a)$
5.  $q(x, p(a), b)$
6.  $p(g(f(a), g(x, f(x))))$
7.  $q(f(a), f(f(x)), f(g(f(z), g(a, b))))$
8.  $r(a, r(a, a))$

**Exercise 1.2** Non Logical symbols:

- constants  $a, b$ ;
- functions  $f_1, g_2$ ;
- predicates  $p_1, r_2, q_3$ ;

Say whether the following strings of symbols are well-formed FOL formulas or terms:

1.  $r(a, g(a, a))$
2.  $g(a, g(a, a))$
3.  $\forall x. \neg p(x)$
4.  $\neg r(p(a), x)$
5.  $\exists a. r(a, a)$

6.  $\exists x.q(x, f(x), b) \rightarrow \forall x.r(a, x)$
7.  $\exists x.p(r(a, x))$
8.  $\forall r(x, a)$

**Exercise 1.3** Non Logical symbols:

- constants  $a, b$ ;
- functions  $f_1, g_2$ ;
- predicates  $p_1, r_2, q_3$ ;

Say whether the following strings of symbols are well-formed FOL formulas or terms:

1.  $a \rightarrow p(b)$
2.  $r(x, b) \rightarrow \exists y.q(y, y, y)$
3.  $r(x, b) \vee \neg \exists y.g(y, b)$
4.  $\neg y \vee p(y)$
5.  $\neg \neg p(a)$
6.  $\neg \forall x.\neg p(x)$
7.  $\forall x \exists y.(r(x, y) \rightarrow r(y, x))$
8.  $\forall x \exists y.(r(x, y) \rightarrow (r(y, x) \vee (f(a) = g(a, x))))$

**Exercise 1.4** Find free variables in the following formulas:

1.  $p(x) \wedge \neg r(y, a)$
2.  $\exists x.r(x, y)$
3.  $\forall x.p(x) \rightarrow \exists y.\neg q(f(x), y, f(y))$
4.  $\forall x \exists y.r(x, f(y))$
5.  $\forall x \exists y.r(x, f(y)) \rightarrow r(x, y)$

**Exercise 1.5** Find free variables in the following formulas:

1.  $\forall x.(p(x) \rightarrow \exists y.\neg q(f(x), y, f(y)))$
2.  $\forall x(\exists y.r(x, f(y)) \rightarrow r(x, y))$
3.  $\forall z.(p(z) \rightarrow \exists y.(\exists x.q(x, y, z) \vee q(z, y, x)))$
4.  $\forall z \exists u \exists y.(q(z, u, g(u, y)) \vee r(u, g(z, u)))$
5.  $\forall z \exists x \exists y(q(z, u, g(u, y)) \vee r(u, g(z, u)))$

## 1.2 Translation

**Exercise 1.6** What is the meaning of the following FOL formulas?

1.  $bought(Frank, dvd)$
2.  $\exists x.bought(Frank, x)$
3.  $\forall x.(bought(Frank, x) \rightarrow bought(Susan, x))$
4.  $\forall x.bought(Frank, x) \rightarrow \forall x.bought(Susan, x)$
5.  $\forall x \exists y.bought(x, y)$

$$6. \exists x \forall y. \text{bought}(x, y)$$

**Exercise 1.7** Which of the following formulas is a formalization of the sentence: "There is a computer which is not used by any student"

1.  $\exists x. (\text{Computer}(x) \wedge \forall y. (\neg \text{Student}(y) \wedge \neg \text{Uses}(y, x)))$
2.  $\exists x. (\text{Computer}(x) \rightarrow \forall y. (\text{Student}(y) \rightarrow \neg \text{Uses}(y, x)))$
3.  $\exists x. (\text{Computer}(x) \wedge \forall y. (\text{Student}(y) \rightarrow \neg \text{Uses}(y, x)))$

**Exercise 1.8** Define an appropriate language and formalize the following sentences using FOL formulas.

1. All Students are smart.
2. There exists a student.
3. There exists a smart student.
4. Every student loves some student.
5. Every student loves some other student.
6. There is a student who is loved by every other student.
7. Bill is a student.
8. Bill takes either Analysis or Geometry (but not both).
9. Bill takes Analysis and Geometry.
10. Bill doesn't take Analysis.
11. No students love Bill.

**Exercise 1.9** Define an appropriate language and formalize the following sentences using FOL formulas.

1. Bill has at least one sister.
2. Bill has no sister.
3. Bill has at most one sister.
4. Bill has (exactly) one sister.
5. Bill has at least two sisters.
6. Every student takes at least one course.
7. Only one student failed Geometry.
8. No student failed Geometry but at least one student failed Analysis.
9. Every student who takes Analysis also takes Geometry.

**Exercise 1.10** Define an appropriate language and formalize the following sentences using FOL formulas.

1. someone likes Mary.
2. nobody likes Mary.
3. nobody loves Bob but Bob loves Mary.
4. if David loves someone, then he loves Mary.
5. if someone loves David, then he (someone) loves also Mary. • everybody loves David or Mary.

**Exercise 1.11** Define an appropriate language and formalize the following sentences using FOL formulas.

1. there is at least one person who loves Mary.
2. there is at most one person who loves Mary.
3. there is exactly one person who loves Mary.
4. there are exactly two persons who love Mary.
5. if Bob loves everyone that Mary loves, and Bob loves David, then Mary doesn't love David.
6. Only Mary loves Bob.

**Exercise 1.12** Define an appropriate language and formalize the following sentences using FOL formulas.

1. "A is above C,D is on E and above F."
2. "A is green while C is not."
3. "Everything is on something."
4. "Everything that is free has nothing on it."
5. "Everything that is green is free."
6. "There is something that is red and is not free."
7. "Everything that is not green and is above B, is red."

**Exercise 1.13** Consider the following sentences:

1. "All actors and journalists invited to the party are late."
2. "There is at least a person who is on time."
3. "There is at least an invited person who is neither a journalist nor an actor."

Formalize the sentences and prove that 3. is not a logical consequence of 1. and 2.