LOI

Basic Notions

Solution **1.1**
Well-formed formulas: 2, 4, 6, and 7
All other strings are NOT well-formed FOL formulas nor terms.

Solution **1.2**
Well-formed formulas: 1, 3, and 6
Well-formed terms: 2
All other strings are NOT well-formed FOL formulas nor terms.

Solution **1.3**
Well-formed formulas: 2, 4, 5, 6, 7, and 8
All other strings are NOT well-formed FOL formulas nor terms.

Solution **1.4**

1. x, y free
2. y free
3. x free
4. no free variables
5. x, y free

Solution **1.5**

1. no free variables
2. y free
3. x free
4. no free variables
5. u free

Translation

Solution **1.6**

1. ”Frank bought a dvd.”
2. ”Frank bought something.”
3. ”Susan bought everything that Frank bought.”
4. ”If Frank bought everything, so did Susan.”
5. ”Everyone bought something.”
6. ”Someone bought everything.”

Solution **1.7** 3

Solution **1.8**

1. $\forall x.(Student(x) \rightarrow Smart(x))$
2. $\exists x.Student(x)$
3. $\exists x.(Student(x) \wedge Smart(x))$
4. $\forall x.(Student(x) \rightarrow \exists y.(Student(y) \wedge Loves(x,y)))$
5. $\forall x.(Student(x) \rightarrow \exists y.(Student(y) \wedge \neg(x=y) \wedge Loves(x,y)))$
6. $\exists x.(Student(x) \wedge \forall y.(Student(y) \wedge \neg(x=y) \rightarrow Loves(y,x)))$
7. $Student(Bill)$
8. $Takes(Bill, Analysis) \leftrightarrow \neg Takes(Bill, Geometry)$
9. $Takes(Bill, Analysis) \wedge Takes(Bill, Geometry)$
10. $\neg Takes(Bill, Analysis)$
11. $\neg\exists x.(Student(x) \wedge Loves(x, Bill))$

Solution **1.9**

1. $\exists x.SisterOf(x, Bill)$
2. $\neg\exists x.SisterOf(x, Bill)$
3. $\forall x\forall y.(SisterOf(x, Bill) \wedge SisterOf(y, Bill) \rightarrow x = y)$
4. $\exists x.(SisterOf(x, Bill) \wedge \forall y.(SisterOf(y, Bill) \rightarrow x = y))$
5. $\exists x\exists y.(SisterOf(x, Bill) \wedge SisterOf(y, Bill) \wedge \neg(x = y))$
6. $\forall x.(Student(x) \rightarrow \exists y.(Course(y) \wedge Takes(x, y)))$
7. $\exists x.(Student(x) \wedge Failed(x, Geometry) \wedge \forall y.(Student(y) \wedge Failed(y, Geometry) \rightarrow x = y))$
8. $\neg\exists x.(Student(x) \wedge Failed(x, Geometry)) \wedge \exists x.(Student(x) \wedge Failed(x, Analysis))$
9. $\forall x.(Student(x) \wedge Takes(x, Analysis) \rightarrow Takes(x, Geometry))$

Solution **1.10** By now you should be able to do it without help.

Solution **1.11** By now you should be able to do it without help.

Solution **1.12**

Language

Constants: A, B, C, D, E, F

Predicates: $On_2, Above_2, Free_1, Red_1, Green_1$

Axioms

1. "A is above C, D is above F and on E." :
   $\phi 1 : Above(A, C) \wedge Above(E, F) \wedge On(D, E)$
2. "A is green while C is not." :
   $\phi 2 : Green(A) \wedge \neg Green(C)$
3. "Everything is on something." :
   $\phi 3 : \forall x\exists y.On(x, y)$
4. "Everything that is free has nothing on it." :
   $\phi 4 : \forall x.(Free(x) \rightarrow \neg\exists y.On(y, x))$
5. "Everything that is green is free." :
   $\phi 5 : \forall x.(Green(x) \rightarrow Free(x))$
6. "There is something that is red and is not free." :
   $\phi 6 : \exists x.(Red(x) \wedge \neg Free(x))$

7. ”Everything that is not green and is above B, is red.” :
   $\phi 7 : \forall x.(\neg Green(x) \wedge Above(x, B) \rightarrow Red(x))$

Solution **1.13**

1. $\forall x.((a(x) \vee j(x)) \wedge i(x) \rightarrow l(x))$
2. $\exists x.\neg l(x)$
3. $\exists x.(i(x) \wedge \neg a(x) \wedge \neg j(x))$

It’s sufficient to find an interpretation I for which the logical consequence does not hold:

	$l(x)$	$a(x)$	$j(x)$	$i(x)$
Bob	F	T	F	F
Tom	T	T	F	T
$Mary$	T	F	T	T