Chapter 1
Reasoning

1.1 Exercises

1.1.1 DPLL Reasoning

Reduction to Conjunctive Normal Form (CNF)

Exercise 1.1 Reduce to Conjunctive Normal Form (CNF) the formula

$$\neg(\neg p \vee q) \vee (r \supset \neg s)$$

Exercise 1.2 Reduce to CNF the formula

$$(\neg p \supset q) \supset (q \supset \neg r)$$

Exercise 1.3 Given the WFF(well-formed-formulas) formula:

$$(A \iff B) \vee C$$

say which of the following WWF formulas are reformulations in CNF of the above formula:

1. $(\neg A \vee B \vee C) \wedge (\neg B \vee A \vee C)$
2. $(C \vee A \vee \neg B \vee C) \wedge (B \vee \neg A \vee C)$
3. $(\neg B \vee A \vee C) \wedge (A \vee B \vee \neg C)$
4. $(B \vee \neg A \vee \neg C) \wedge (\neg B \vee A \vee C)$

Exercise 1.4 Given the formula:

$$(\neg A \vee B \vee D) \wedge (A \vee \neg C) \wedge (D \vee C)$$

which of the following sequences of literal assignments could be generated by DPLL? The assignments are shown in order: then

    C, D, A

means: first C, then D, then A. Choose one or more of the following:

  1. D, ¬ C
  2. C, A,B
  3. C,A, D
  4. C, A, ¬B
  5. C, B, A

Exercise 1.5 Compute the CNF of the formula below

$$(q \wedge p) \vee \neg p$$

Exercise 1.6 Compute the CNF of the formula below

$$(p \supset q) \equiv (\neg q \supset \neg p)$$

Exercise 1.7 Compute the CNF of the formula below

$$(p \wedge r) \supset q$$

Exercise 1.8 Convert a formula in CNF: (A ∨ B) ⊃ ¬A

Exercise 1.9 Convert a formula in CNF: (C ⊃ ¬A) ∧ ¬(B ∧ ¬A)

Exercise 1.10 Indicate which of the following formulas in Logic of Propositions ($\mathcal{LOP}$) are not in CNF.

  1. (X ∨ Z) ∧ (¬X ∨ Y) ∧ (¬X ∧ Y ∨ ¬Z)
  2. (X ∧ Z) ∨ (¬X ∧ Y) ∨ (¬X ∧ Y ∧ ¬Z)
  3. X ∧ (¬X ∨ Y) ∧ (¬X ∨ Y ∨ ¬Z) ∧ Z

Check Satisfiability via CNF

Exercise 1.11 Check the satisfiability of the following formula

$$(\neg p \vee q) \wedge (\neg r \vee q)$$

Exercise 1.12 Check the satisfiability of the following formula

$$(p \vee q) \wedge (p \vee \neg p) \wedge (\neg q \vee q) \wedge (\neg q \vee \neg p) \wedge (\neg q \vee \neg p) \wedge (\neg q \vee \neg p) \wedge (\neg q \vee q) \wedge (p \vee \neg p) \wedge (p \vee q)$$

Exercise 1.13 Check the satisfiability of the following formula

$$(q \vee \neg p) \wedge (q \vee \neg p)$$

Exercise 1.14 Check the satisfiability of the following formula

$$(q \vee \neg p) \wedge (\neg q \vee p) \wedge (p \vee q)$$

Exercise 1.15 (Unit Propagation) Consider the following CNF formula ϕ, taking $\phi = \{\{p\}, \{\neg p, \neg q\}, \{\neg q, r\}\}$. Check whether ϕ is satisfiable by unit propagation. If so, find an interpretation I so that $I \models \phi$.

Exercise 1.16 (Unit Propagation) Consider the following CNF formula

$$\phi = \{\{p\}, \{\neg p\}, \{\neg q, r\}\}$$

is satisfiable and if so, find an interpretation / so that $I \models \phi$.

Exercise 1.17 (Use DPLL to prove satisfiability) Check satisfiability of: B $\wedge$ ¬A $\wedge$ (¬C $\vee$ A) $\wedge$ (B $\vee$ C). In terms of clauses is: B, ¬A, ¬C, A, B, C

Exercise 1.18 (Use DPLL to prove satisfiability) Check satisfiability of: (C ⊃ A) $\wedge$ (C ⊃ B) $\wedge$ ¬(A $\wedge$ B).

Exercise 1.19 (Use DPLL to prove satisfiability) Check satisfiability of the clause: ¬A, C, D, ¬B, F, D, ¬B, ¬F, ¬C, ¬D, ¬B, B, ¬C, ¬A, B, F, C, B, ¬F, ¬D, A, E, A, F, ¬F, C, ¬E, A, ¬C, ¬E.

Exercise 1.20 (Use DPLL to prove satisfiability) By using DPLL, prove the unsatisfiability of (B ⊃ A) $\wedge$ (¬A $\wedge$ B).

Exercise 1.21 (Use DPLL to prove satisfiability) By using DPLL, prove the validity of (A ⊃ B) ⊃ (¬B ⊃ ¬A).

Exercise 1.22 (Use DPLL to prove satisfiability) Given the formula P in CNF {{A,B,C}, {{A,B,¬C}, {A,¬B,C}, {A,¬B,¬C}, {¬A,D}, {A,¬D,¬E,F}, {¬A,G}}, indicate which of the following statements are true (one or more):

1. The formula P is satisfiable
2. The formula P is not satisfiable
3. ¬E,F,G,D,A is a possible sequence of assignments generated by the procedure
4. F, G, ¬E, D, A is a possible sequence of assignments generated by the procedure
5. D, A, ¬E, F, G is a possible sequence of assignments generated by the procedure