Reasoning

DPLL Reasoning

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Reduction to Conjunctive Normal Form (CNF)

Solution 1.1

1. $\neg(\neg p \vee q) \vee (\neg r \vee \neg s)$
2. $(\neg\neg p \wedge \neg q) \vee (\neg r \vee \neg s)$
3. $(p \wedge \neg q) \vee (\neg r \vee \neg s)$
4. $(p \vee \neg r \vee \neg s) \wedge (\neg q \vee \neg r \vee \neg s)$

Solution 1.2

1. $\neg(\neg p \supset q) \vee (q \supset \neg r)$
2. $\neg(p \vee q) \vee (\neg q \vee \neg r)$
3. $(\neg p \wedge \neg q) \vee (\neg q \vee \neg r)$
4. $(\neg p \vee \neg q \vee \neg r) \wedge (\neg q \vee \neg r)$

Solution 1.3

- $(\neg A \vee B \vee C) \wedge (\neg B \vee A \vee C)$
- $(C \vee A \vee \neg B \vee C) \wedge (B \vee \neg A \vee C)$

Solution 1.4

- D,¬C. This choice is correct. There are no unit clauses. D is pure. When D simplified A and ¬C are equally valid solutions.
- C,A,B. This choice is wrong as C is not pure
- C,A,D. See above.
- C,A,¬B. See above.
- C,B,A. See above.

Solution 1.5

$$
\begin{array}{r}
CNF((q \wedge p) \vee \neg p) \\
CNF((q \wedge p)\ ) \otimes CNF(\ \neg p) \\
(CNF(q\ ) \wedge CNF(\ p)) \otimes \neg p \\
(q \wedge p) \otimes \neg p \\
(q \vee \neg p) \wedge (p \vee \neg p)
\end{array}
$$

Solution 1.6

$CNF((p \supset q) \equiv (\neg q \supset \neg p))$
$CNF((p \supset q) \supset (\neg q \supset \neg p)) \wedge CNF((\neg q \supset \neg p) \supset (p \supset q))$
$(CNF((\neg(p \supset q)) \otimes CNF((\neg q \supset \neg p))) \wedge (CNF((\neg(\neg q \supset \neg p)) \otimes CNF((p \supset q)))$
$(CNF(p) \wedge CNF(\neg q)) \otimes (CNF(q) \otimes CNF(\neg p)) \wedge (CNF((\neg q)) \wedge CNF(p)) \otimes$
$(CNF(\neg p) \otimes CNF(q))$
$(p \wedge \neg q) \otimes (q \otimes \neg p) \wedge (\neg q \wedge p) \otimes (\neg p \otimes q)$
$(p \wedge \neg q) \otimes (q \wedge \neg p) \wedge (\neg q \wedge p) \otimes (\neg p \wedge q)$
$(p \vee q) \wedge (p \vee \neg p) \wedge (\neg q \vee q) \wedge (\neg q \vee \neg p) \wedge (\neg q \vee \neg p) \wedge (\neg q \vee \neg p) \wedge (\neg q \vee q) \wedge (p \vee \neg p) \wedge (p \vee q)$

Solution **1.7**

$$\begin{aligned} &CNF((p \wedge r) \supset q) \\ &CNF((\neg(p \wedge r)) \otimes CNF(q) \\ &(CNF((\neg p)) \otimes CNF(\neg r)) \otimes q \\ &(\neg p \otimes \neg r) \otimes q \\ &(\neg p \vee \neg r) \otimes q \\ &(\neg p \vee \neg r \vee q) \end{aligned}$$

Solution **1.8**

$$\begin{aligned} &CNF(\neg(A \vee B)) \otimes CNF(\neg A) \\ &(CNF(\neg A) \wedge CNF(\neg B)) \otimes CNF(\neg A) \\ &(\neg A \wedge \neg B) \otimes \neg A \\ &(\neg A \vee \neg A) \wedge (\neg B \vee \neg A) \\ &\neg A \wedge (\neg B \vee \neg A) \end{aligned}$$

In terms of clauses becomes ¬ A, ¬ B ,¬ A

Solution **1.9**

$$\begin{aligned} &CNF(C \supset \neg A) \wedge CNF(\neg(B \wedge \neg A)) \\ &(CNF(\neg C) \otimes CNF(\neg A)) \wedge (CNF(\neg B) \otimes CNF(\neg\neg A)) \\ &(\neg C \vee \neg A) \wedge (\neg B \vee A) \end{aligned}$$

In terms of clauses becomes ¬ C,¬ A, ¬ B, A

Solution **1.10** The correct answers are (1) and (2). By definition, only (3) is in CNF. (1) is not because there is also a ∧ in the last clause. (2) is not because it is a disjunction of conjunctions.

Check satisfiability via CNF

Solution **1.11** Here is the solution by using DPLL:

1. (¬ p ∨ q) ∧ (¬ r ∨ q)
2. {{¬p, q}, {¬r, q} }
3. $\{\{\neg p, \top\}, \{\neg r, \top\}\}|_q$
4. { }

Solution **1.12** Here is the solution by using DPLL:

1. $(p \vee q) \wedge (p \vee \neg p) \wedge (\neg q \vee q) \wedge (\neg q \vee \neg p) \wedge (\neg q \vee \neg p) \wedge (\neg q \vee \neg p) \wedge (\neg q \vee q) \wedge (p \vee \neg p) \wedge (p \vee q)$
2. $\{\{p, q\}, \{p, \neg p\}, \{\neg q, q\}, \{\neg q, \neg p\}, \{\neg q, \neg p\}, \{\neg q, \neg p\}, \{\neg q, q\}, \{p, \neg p\}, \{p, q\}\}$
3. $\{\{\top, q\}, \{\top, \bot\}, \{\neg q, q\}, \{\neg q, \bot\}, \{\neg q, \bot\}, \{\neg q, \bot\}, \{\neg q, q\}, \{\top, \bot\}, \{\top, q\}\}|_p$
4. $\{\{\neg q, q\}, \{\neg q\}, \{\neg q\}, \{\neg q\}, \{\neg q, q\}\}$
5. $\{\{\top, \bot\}, \{\top\}, \{\top\}, \{\top\}, \{\top, \bot\}\}|_{\neg q}$
6. $\{\}$

Solution **1.13** Here is the solution by using DPLL:

1. $(q \vee \neg p) \wedge (q \vee \neg p)$
2. $\{\{q, \neg p\}, \{q, \neg p\}\}$
3. $\{\{q, \top\}, \{q, \top\}\}|_{\neg p}$
4. $\{\}$

Solution **1.14** Here is the solution by using DPLL:

1. $(q \vee \neg p) \wedge (\neg q \vee p) \wedge (p \vee q)$
2. $\{\{q, \neg p\}, \{\neg q, p\}, \{p, q\}\}$
3. $\{\{q, \top\}, \{\neg q, \bot\}, \{\bot, q\}\}|_{\neg p}$
4. $\{\{\neg q\}, \{q\}\}$
5. $\{\{\bot\}, \{\top\}\}|_q$
6. $\{\{\}\}$

Solution **1.15**

$\{\{p\}, \{\neg p, \neg q\}, \{\neg q, r\}\}$
$\{\{p\}, \{\neg p, \neg q\}, \{\neg q, r\}\}\ |_p$
$\{\{\top\}, \{\bot, \neg q\}, \{\neg q, r\}\}$
$\{\{\neg q\}, \{\neg q, r\}\}$
$\{\{\neg q\}, \{\neg q, r\}\}\ |_{\neg q}$
$\{\{\top\}, \{\top, r\}\}$
$\{\}$

ϕ is satisfiable, and $I = \{p, \neg q\}$. The literal r is left undefined, because in order to satisfy the formula there is no need to evaluate it. This is an example of partial evaluation.

Solution **1.16**

$\{\{p\}, \{\neg p\}, \{\neg q, r\}\}$
$\{\{p\}, \{\neg p\}, \{\neg q, r\}\}|_p$
$\{\{\top\}, \{\bot\}, \{\neg q, r\}\}$
$\{\{\}, \{\neg q, r\}\}$
$\{\ldots, \{\}, \ldots\}$

The second clause cannot be verified, and thus the entire formula is not satisfiable.

Solution **1.17**

$\{\{\mathrm{B}\}, \{\neg\mathrm{A}\}, \{\neg\mathrm{C}, \mathrm{A}\}, \{\mathrm{B}, \mathrm{C}\}\}$

{{⊤}, {¬A}, {¬C, A}, {⊤, C}}
{{¬A}, {¬C, A}}
{{⊤}, {¬C, ⊥}}
{{¬C}}
{{⊤}}
{}

Therefore the formula is satisfiable. A possible model M = ¬A, B, ¬C.

Solution **1.18** First convert it in CNF: (¬ C ∨ A) ∧ (¬ C ∨ B) ∧ (¬ A ∨ ¬ B)
In terms of clauses is: {{¬C, A}, {¬C, B}, {¬A, ¬B}}
We can apply the pure literal:

{{¬C, A}, {¬C, B}, {¬A, ¬B}}
{{⊤, A}, {⊤, B}, {¬A, ¬B}}
{{¬A, ¬B}}

We then select a literal for the splitting rule:

{{¬A}, {¬A, ¬B}}
{{⊤}, {⊤, ¬B}}
{}

Therefore the formula is satisfiable.

Solution **1.19** There are no unit clauses, nor pure literals. Let us then select the literal A and apply the splitting rule.

{{A}, {¬A, C, D}, {¬B, F, D}, {¬B, ¬F, ¬C}, {¬D, ¬B}, {B, ¬C, ¬A}, {B, F, C}, {B, ¬F, ¬D}, {A, E}, {A, F}, {¬F, C, ¬E}, {A, C, ¬E}}
{{⊤}, {⊥, C, D}, {¬B, F, D}, {¬B, ¬F, ¬C}, {¬D, ¬B}, {B, ¬C, ⊥}, {B, F, C}, {B, ¬F, ¬D}, {⊤, E}, {⊤, F}, {¬F, C, ¬E}, {⊤, C, ¬E}}
{{C, D}, {¬B, F, D}, {¬B, ¬F, ¬C}, {¬D, ¬B}, {B, ¬C}, {B, F, C}, {B, ¬F, ¬D}, {E}, {F}, {¬F, C, ¬E}, {C, ¬E}}
{{C, D}, {¬B, F, D}, {¬B, ¬F, ¬C}, {¬D, ¬B}, {B, ¬C}, {B, F, C}, {B, ¬F, ¬D}, {⊤}, {F}, {¬F, C, ⊥}, {C, ⊥}}
{{C, D}, {¬B, F, D}, {¬B, ¬F, ¬C}, {¬D, ¬B}, {B, ¬C}, {B, F, C}, {B, ¬F, ¬D}, {F}, {¬F, C}, {C}}
{{C, D}, {¬B, ⊤, D}, {¬B, ⊥, ¬C}, {¬D, ¬B}, {B, ¬C}, {B, ⊤, C}, {B, ⊥, ¬D}, {⊤}, {⊥, C}, {C}}
{{C, D}, {¬B, ¬C}, {¬D, ¬B}, {B, ¬C}, {B, ¬D},{C}, {C}}
{{⊤, D}, {¬B, ⊥}, {¬D, ¬B}, {B, ⊥}, {B, ¬D}, {⊤}, {⊤}}
{{¬B}, {¬D, ¬B}, {B}, {B, ¬D}}
{{⊤}, {¬D, ⊤}, {⊥}, {⊥, ¬D}}
{{¬D}, {}, {¬D}}

There is an empty clause. Therefore it returns false. We need to check with the literal ¬A:

{{¬A}, {¬A, C, D}, {¬B, F, D}, {¬B, ¬F, ¬C}, {¬D, ¬B}, {B, ¬C, ¬A}, {B, F, C}, {B, ¬F, ¬D}, {A, E}, {A, F}, {¬F, C, ¬E}, {A, C, ¬E}}

Solution 1.20 First convert it in CNF: $(\neg B \vee A) \wedge \neg A \wedge B$
In terms of clauses is: $\{\{\neg B, A\}, \{\neg A\}, \{B\}\}$

$\{\{\neg B, A\}, \{\neg A\}, \{B\}\}$
$\{\{\neg B, \bot\}, \{\top\}, \{B\}\}$
$\{\{\neg B\}, \{B\}\}$
$\{\{\top\}, \{\bot\}\}$
$\{\{\}\}$

There is an empty clause, therefore it returns false.
Therefore the formaula is unstatistiable.

Solution 1.21 First negate the formula: $\neg((A \supset B) \supset (\neg B \supset \neg A))$
Then convert it in CNF: $(\neg A \vee B) \wedge \neg B \wedge A$
In terms of clauses is: $\{\{\neg A, B\}, \{\neg B\}, \{A\}\}$

$\{\{\neg A, B\}, \{\neg B\}, \{A\}\}$
$\{\{\neg A, \bot\}, \{\top\}, \{A\}\}$
$\{\{\neg A\}, \{A\}\}$
$\{\{\}\}$

There is an empty clause, therefore it returns false.
Therefore the formaula is valid.

Solution 1.22 Applying the algorithm we observe that there are no unit clauses. There are, however, pure literals. Consequently, we must necessarily start from one among ¬E, F and G, not necessarily in that same order:

$$\{\{A,B,C\},\{A,B,\neg C\},\{A,\neg B,C\},\{A,\neg B,\neg C\},\{\neg A,D\},\{A,\neg D,\neg E,F\},\{\neg A,G\}\}$$

Since ¬E and F are in the same clause, only one of the two can be applied. Below are the clauses obtained by choosing G and F (or G and ¬E):

$$\{\{A,B,C\},\{A,B,\neg C\},\{A,\neg B,C\}, \{A,\neg B,\neg C\},\{\neg A,D\}\}$$

We observe that there are no unit clauses. This time D is the only pure literal.

$$\{\{A,B,C\},\{A,B,\neg C\},\{A,\neg B,C\}, \{A,\neg B,\neg C\}\}$$

We observe that there are no unit clauses. A is the only pure literal.

It reduces to { }. Consequently, the algorithm returns true.

Thus the formula is satisfiable and therefore (1) is true, while (2) is false.

From the application of the steps, we also see that none of the given sequences are possible, consequently (3), (4) and (5) are also false. In fact, ¬E and F cannot be there at the same time.